# COMPARATIVE STUDY OF THIRD GRADE BLOOD FLOW THROUGH STENOSED ARTERY WITH CONSTANT AND VARIABLE VISCOSITIES.

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**ABSTRACT**

*Mathematical models for the comparative study of blood flow through stenosed artery with constant viscosity and variable viscosity dependent on red blood cells concentration (hematocrit) taking into consideration the externally applied magnetic field and slip velocity is presented. The laminar, incompressible, fully developed, non-Newtonian (third grade) flow of blood in an artery having stenosis is numerically studied. Effect of variable viscosity, slip velocity and magnetic field in the blood flow with constant and variable viscosities are discussed of analytically and graphically. All the flow characteristics are established to be effected due to the combined effects of the variable viscosity, magnetic field and slip velocity. Analytical expression for both blood flow models with constant and variable viscosities for the velocity profiles, volumetric flow rate, shear stress and resistance to flow are derived. The study provides an insight into the effects of variable viscosity, magnetic and slip velocity on the velocity profiles, volumetric flow rate, shear stress and resistance to flow on blood flow models with constant and variable viscosities.*

**Keywords:** *Constant Viscosity, Variable Viscosity, Stenosed Artery, Slip Velocity, Magnetic Field, third grade, Volumetric flow rate, hematocrit, Shear stress, resistance to flow*

## 1.0 INTRODUCTION

Atherosclerosis which is one of the cardiovascular diseases have been responsible for many deaths in both developed and developing countries. Blood which is composed of red blood cells, white blood cells, plasma, platelet is considered to be one of the most important multi-component mixtures. One of the causes of circulating disorders which can in effect lead to occluding the blood supply is the presence of stenosis in the cardiovascular system. The occlusion of normal blood supply can cause a serious consequence such as myocardial infarction and cerebral stroke.

Several researchers had carried out investigation and explored the behavior of blood flow with constant viscosity under the influence of magnetic field or slip velocity or both. Magnetic field influence on the pulsatile flow of biofluid was studied by Alimohamadi *et al* [1], Das and Sahs [2], used finite Hankel and Laplace transforms to obtained analytical solution to pulsatile flow of blood through a stenosed porous medium with periodic body acceleration and under the influence of magnetic fields. The Pulsatile flow of blood through narrow arteries with axisymmetric mild stenosis and also with the effect of magnetic field was investigated by Sankar and Lee [3], Bali and Awasthi [4] examined the effect of externally imposed uniform magnetic field on the nonlinear casson flow field on the multi stenosed artery with core region. They modelled blood as a casson fluid by properly accounting for yield stress of blood in small blood vessel. Singh and Singh [5] studied the effect of an externally applied uniform magnetic field on the axially non-symmetric but radially symmetric atherosclerotic artery with core region. Haleh, *et al* [6] considered the non- Newtonian blood flow in the stenosed artery in the presence of magnetic field. They used magnetic field to destroy the created vortex after the stenosis region corresponds to non-uniforming of flow in this region.

All the aforementioned researchers did not consider the slip effect. In line with this, Amit and Shrivaster [7] study the flow of blood in a multiple stenosed artery employing velocity slip conditions under the externally applied transverse magnetic field. They modelled blood as Herschel Bulkley fluid to represent the non-Newtonian character of the blood in small blood vessel. Raja and Varshney [8] developed a mathematical model to study the MHD oscillatory blood flow through stenosed artery under the effect of slip velocity. They assumed blood to be Newtonian. A theoretical investigation concerning the influence of externally imposed arterial segment by taking into account the slip velocity at the wall of the artery has been investigated by Singh *et al* [9]. They used perturbation to solve the couple implicit system of nonlinear differential equations that govern the flow of blood. Their results show that the flow is appreciably influenced by slip velocity in the presence of the periodic body acceleration. Other researchers that considered slip velocity in their studies includes: Arun [10], Verma *et al* [11], Reddy *et al* [12], Bhatnagar and Strivastava [13], Guar and Gupta [14].

The above researchers considered only constant viscosity in their studies. Some of the researchers that considered variable viscosity in their studies are: Sanjeev and Chandrashekhar [15], Chitra and Karthikeyan [16], Singh and Rathee [17], Jagdish and Rajbala [18].

In this study, mathematical models are proposed to describe the blood flow through stenosed artery with constant viscosity and variable viscosity dependent on red blood concentration. Incorporated into the models are the externally applied magnetic field and slip velocity.

**2.0 Mathematical Models**

The momentum equations describing the steady fluid flow models with constant viscosity and variable viscosity as obtained by Mohammed [19] and Jimoh [20] are respectively given as

$$\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2-\frac{\partial \hat{P}}{\rho\partial z}-\frac{\sigma\beta_0^2 w}{\rho}=0 \qquad (2.1)$$

and

$$\frac{\mu_0}{\rho}\left[1+N_1\left(1-\left(\frac{r}{R_0}\right)^m\right)\right]\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^3-\frac{\partial \hat{P}}{\rho\partial z}-\frac{\sigma\beta_0^2 w}{\rho}=0 \qquad (2.2)$$

As a result of the constricted artery in the stenotic region as shown in figure 1, one employed slip velocity so that the corresponding slip conditions to (2.1) and (2.2) are respectively given as

$$\left.\begin{aligned} &w = w_s \quad \text{at} \quad r = R(z) \\ &\frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0 \end{aligned}\right\} \qquad (2.3)$$

and

$$\left.\begin{aligned} &w = w_{Ns} \quad \text{at} \quad r = R(z) \\ &\frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0 \end{aligned}\right\} \qquad (2.3)$$

To non-dimensionalize equations (2.1), (2.2), (2.3) and (2.4), we introduce the following parameters and variables

$$\left.\begin{aligned} &\bar{w} = \frac{w}{d/t_0}, \qquad y = r/R_0 \\ &V_0 = \frac{w_s t_0}{d}, \quad V_{0N} = \frac{w_{Ns} t_0}{d} \end{aligned}\right\} \qquad (2.4)$$

When equation (2.4) is substituted into (2.1) and (2.2), after simplifying one obtain respectively

$$\frac{1}{RE}\cdot\frac{\partial}{\partial y}\left(y\frac{\partial \bar{\omega}}{\partial y}\right)+\Omega\left(6\left(\frac{\partial \bar{\omega}}{\partial y}\right)^2\frac{\partial^2 \bar{\omega}}{\partial y^2}+\frac{2}{y}\left(\frac{\partial \bar{\omega}}{\partial y}\right)^3\right)+G-M\bar{\omega}=0 \qquad (2.5)$$

as the dimensionless momentum equation for the blood flow with constant viscosity.

and

$$\frac{1}{RE_N}\frac{1}{y}\frac{\partial}{\partial y}\left[y\left(1+N(1-y^m)\right)\frac{\partial \bar{w}}{\partial y}\right]+\Omega_N\left(6\left(\frac{\partial \bar{w}}{\partial y^2}\right)^2\frac{\partial^2 \bar{w}}{\partial y^2}+\frac{2}{y}\left(\frac{\partial \bar{w}}{\partial y}\right)^3\right)+$$

$$+G_N-M_N\bar{w}=0 \qquad (2.6)$$

as the dimensionless momentum equation for the blood flow with variable viscosity.

The corresponding dimensionless slip conditions to (2.5) and (2.6) can be simplified respectively as

$$\left.\begin{aligned} &\bar{w} = V_0 \quad \text{at} \quad y = \frac{R(z)}{R_0} = R_b \\ &\frac{\partial \bar{w}}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{aligned}\right\} \qquad (2.7)$$

and

$$\left.\begin{aligned} &\bar{w} = V_{0N} \quad \text{at} \quad y = R_b \\ &\frac{\partial \bar{w}}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{aligned}\right\} \qquad (2.8)$$

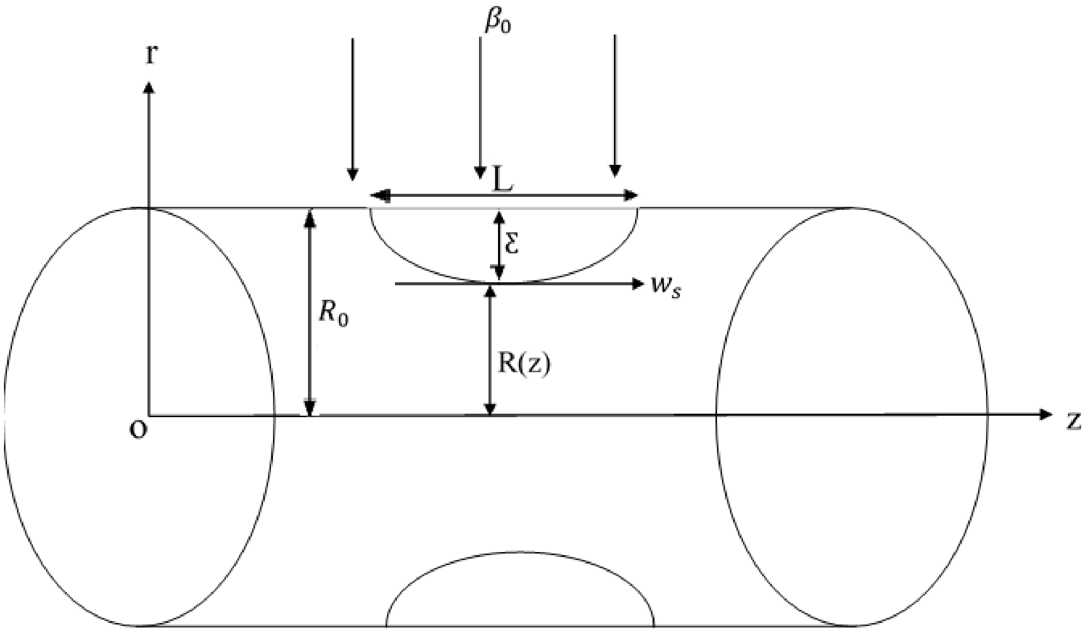


Figure1.Geometry of the stenosis

and has been described by Young [21] and Biswas [22]

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\varepsilon}{2R_0}\left[1 + \frac{\cos\pi z}{L}\right], & \text{for } |z| \leq L \\ R_0, & \text{for } |z| > L \end{cases} \qquad (2.9)$$

### 3.0 Methods of Solution

To obtain solution to (2.5) using Galerkin weighted residual method, one assumes a trial function of the form

$$\bar{w}(y) = a_0 + a_1 y + a_2 y^2 \qquad (3.1)$$

Subjecting (3.1) to the slip conditions (2.7) and after simplification yields

$$\bar{w}(y) = \frac{V_0 y^2}{Rb^2} + a_0\left(1 - \frac{y^2}{Rb^2}\right) + a_2 y^2\left(1 - \frac{y^2}{Rb^2}\right) \qquad (3.2)$$

Using the transformation $\bar{r} = \frac{y}{Rb}$ in (3.2), after simplifying and dropping the bar one obtain

$$w(r) = V_0 r^2 + a_0(1 - r^2) + a_2 r^2(1 - r^2) \qquad (3.3)$$

The residue for equation (2.5) can be written as

$$R_1(a_0, a_2, r) = G + \frac{1}{RE}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \Omega\left(6\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - Mw \qquad (3.4)$$

Substituting (3.3) into (3.4) and simplifying in full to obtain

$$R_1(a_0, a_2.r) = G + \frac{1}{RE}(4V_0 + 4a_2 - 4a_0 - 16a_2r^2) + \Omega(48V_0^2r^2 - 144V_0^2a_0r^2 + 144V_0^2a_2r^2 - 480V_0^2a_2r^4 + 144V_0a_0^2r^2 - 40V_0a_0a_2r^2 + 960V_0a_0a_2r^2 + 144V_0a_2^2r^2 - 144V_0a_2^2r^4 + 1344V_0a_2^2r^6 + 144a_0^2a_2r^2 - 480a_0^2a_2r^4 - 144a_0a_2^2r^2 + 960a_0a_2^2r^4 - 1344a_0a_2^2r^6 - 480a_2^3r^4 + 1344a_2^3r^6 - 48a_0^3r^2 + 48a_2^3r^2 - 1152a_2^3r^8 - 128a_2^3r^8 + 192V_0a_2^2r^6 - 192a_0a_2^2r^6 + 192a_2^3r^6 - 96V_0^2a_2r^4 + 192V_0a_0a_2r^4 - 192V_0a_2^2r^4 - 96a_0^2a_2r^4 + 192a_0a_2^2r^4 - 96a_2^3r^4 + 16V_0^3r^2 - 48V_0^2a_0r^2 + 48V_0^2a_2r^2 + 48V_0a_0^2r^2 - 96V_0a_0a_2r^2 + 48V_0a_2^2r^2 - 16a_0^3r^2 + 48a_0^2a_2r^2 - 48a_0a_2^2r^2 + 16a_2^3r^2) - M(V_0r^2 + a_0 - a_0r^2 + a_2r^2 - a_2r^4) \qquad (3.5)$$

By differentiating (3.3) with respect to $a_0$ *and* $a_2$, one obtain $(1 - r^2)$ and $r^2(1 - r^2)$ respectively as the weight functions.

By taking the orthogonality of the residue $R_1(a_0, a_2, r)$ with respect to the weight functions $(1 - r^2)$ and $r^2(1 - r^2)$ one obtain the following systems of nonlinear equation;

$$327520\Omega a_2^3 - 29568\Omega a_0^3 + 191136\Omega V_0 a_2^2 + 88704\Omega V_0 a_0^2 - 280896\Omega a_2^2 a_0 - 62304\Omega a_0^2 a_2 - 341088\Omega V_0 a_0 a_2 - \left(1848M + 88704\Omega V_0^2 + \frac{9240}{RE}\right) a_0 + \left(-264M + 25344\Omega V_0^2 - \frac{44352}{RE}\right) a_2 = -2310G - \frac{9240V_0}{RE} - 22176\Omega V_0^2 + 462MV_0 \quad (3.6)$$

$$-164736\Omega a_0^3 - 6720\Omega a_2^3 + 4944208\Omega V_0 a_0^2 + 1331616\Omega V_0 a_2^2 - 244608\Omega a_0 a_2^2 - 329472\Omega a_2 a_0^2 + 1194336\Omega V_0 a_0 a_2 - \left(3432M + 494208\Omega V_0^2 + \frac{24024}{RE}\right) a_0 - \left(1144M + 329472\Omega V_0^2 - \frac{54912a_2}{RE}\right) a_2$$

$$= -6006G - \frac{24024V_0}{RE} - 123552\Omega V_0^2 + 2574MV_0 \quad (3.7)$$

By substituting the appropriate values of the parameters $\Omega, V_{0,}\ M, G, and\ RE$ into (3.6) and (3.7) and solving the system of nonlinear equations, one obtain the values for $a_0\ and\ a_2$ which when substituted into (3.3), the velocity profiles were obtained which are shown in table 1.

Similarly, to obtain solution to equation (2.6) following the same procedure as indicated above, one can write residue for (2.6) as

$$R_2(a_0, a_2, r) = G_N + \frac{1}{RE_N}\left(1 + N(1 + r^m)\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \Omega_N\left(6\left(\frac{\partial w}{\partial r^2}\right)^2\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - M_N w \quad (3.8)$$

The following systems of nonlinear equation were obtained by following the same step just like in the previous section

$$14784V_{0N}^3 RE_N\Omega_N - 44352RE_N V_{0N}^2\Omega_N a_0 - 12672RE_N V_{0N}^2\Omega_N a_2 + 44352RE_N V_{0N}\Omega_N a_0^2 + 25344RE_N V_{0N}\Omega_N a_0 a_2 + 14784RE_N V_{0N}\Omega_N a_2^2 - 14784RE_N\Omega_N a_0^3 - 12672RE_N\Omega_N a_0^2 a_2 - 1478RE_N\Omega_N a_0 a_2^2 - 2560RE_N\Omega_N a_2^3 - 231M_N RE_N V_{0N} - 924M_N RE_N a_0 - 132M_N RE_N a_2 + 1155G_N RE_N + 3696NV_{0N} - 3696Na_0 + 1584Na_2 + 4620V_{0N} - 4620a_0 + 924a_2 = 0 \quad (3.9)$$

$$82368RE_N V_{0N}^3\Omega_N - 247104RE_N V_{0N}^2\Omega_N a_0 - 164736RE_N V_{0N}^2\Omega_N a_2 + 247104RE_N V_{0N}\Omega_N a_0^2 + 32472RE_N V_{0N}\Omega_N a_0 a_2 + 122304RE_N V_{0N}\Omega_N a_2^2 - 82364RE_N\Omega_N a_0^3 - 164736RE_N\Omega_N a_0^2 a_2 - 122304RE_N\Omega_N a_0 a_2^2 - 33792RE_N\Omega_N a_2^3 - 1287M_N RE_N V_{0N} - 1716M_N RE_N a_0 - 572M_N RE_N a_2 + 3003G_N RE_N + 6864NV_{0N} - 6864Na_0 - 2288Na_2 + 12012V_{0N} - 12012a_0 - 8580a_2 = 0 \quad (3.10)$$

Substituting the appropriate values of the parameters $RE_N$, $V_{0N}$, $\Omega_N$, $M_N$, and $G_N$ into (3.9) and (3.10) and solving the system of nonlinear equations, one obtain the values for $a_0\ and\ a_2$ which are shown in table 2.

**Volume Flow Rate**

The volume flow rate denoted by Q can be simplified as

$$Q = 12\left[3V_0(R(z))^4 + a_0\left(6(R(z))^2 - 3(R(z))^4\right) + a_2\left(3(R(z))^4 - 2(R(z))^6\right)\right] \quad (3.11$$

**Shear Stress**

The shear stress denoted by $\tau_S$ can be simplified as

$$\tau_s = 2\mu R(Z)\left(V_0 - a_0 + a_2 - 2R((Z))^2 a_2\right) + 16R(Z)\beta_3(V_0 - a_0 + a_2 - 2(R(Z))^2 a_2) \quad (3.12)$$

**Resistance to Flow**

The resistance to flow can be denoted as ψ can be simplified as

$$\psi = \frac{-\frac{\partial \bar{P}}{\partial z}}{12\left[3V_0(R(z))^4 + a_0\left(6(R(z))^2 - 3(R(z))^4\right) + a_2\left(3(R(z))^4 - 2(R(z))^6\right)\right]} \quad (3.13)$$

**Table 1: Values of the parameters used in the numerical results and the corresponding Velocity profile for the blood flow with Constant Viscosity.**

| Figures | G | $V_0$ | RE | $\Omega$ | M | w(r) |
|---|---|---|---|---|---|---|
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | $0.5746 – 0.3755r^2 – 0.0041r^2(1-r^2)$ |
| 2a | 2.0 | 0.25 | 0.9 | 10 | 0.35 | $0.7056 – 0.5056r^2 – 0.0473r^2(1-r^2)$ |
| | 2.5 | 0.25 | 0.9 | 10 | 0.35 | $0.7489 – 0.5489r^2 – 0.0670r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | $0.3188 – 0.1188r^2 – 0.0169r^2(1-r^2)$ |
| 3a | 1.5 | 0.25 | 0.9 | 10 | 0.65 | $0.2850 – 0.0850r^2 – 0.0009r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.95 | $0.2517 – 0.0517r^2 – 0.0058r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | $0.5769 – 0.3269r^2 – 0.1192r^2(1-r^2)$ |
| 4a | 1.5 | 0.35 | 0.9 | 10 | 0.35 | $0.6744 – 0.3244r^2 – 0.1179r^2(1-r^2)$ |
| | 1.5 | 0.45 | 0.9 | 10 | 0.35 | $0.7718 – 0.3218r^2 – 0.1167r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | $0.3385 – 0.1385r^2 – 0.0483r^2(1- r^2)$ 0.3539 – |
| 5a | 1.5 | 0.25 | 0.9 | 20 | 0.35 | $0.1539r^2 – 0.0526r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 30 | 0.35 | $0.3823 – 0.1823r^2 – 0.0558r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.3 | 10 | 0.35 | $0.2669 – 0.0669r^2 – 0.0028r^2(1-r^2)$ |
| 6a | 1.5 | 0.25 | 0.6 | 10 | 0.35 | $0.3118 – 0.1118r^2 – 0.0179r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | $0.3338 – 0.1375r^2 – 0.0333r^2(1-r^2)$ |

**Table 2: Values of the parameters used in the numerical results and the corresponding Velocity profile for the blood flow with Variable Viscosity.**

| Figs | $G_N$ | $V_{0N}$ | $RE_N$ | $\Omega_N$ | $M_N$ | N | w(r) |
|---|---|---|---|---|---|---|---|
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.2582 – 0.0582r^2 – 0.0038r^2(1-r^2)$ |
| 2b | 2.0 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.3065 – 0.1065r^2 – 0.0069r^2(1-r^2)$ |
| | 2.5 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.3438 – 0.1438r^2 – 0.0210 r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.3342 – 0.1342r^2 – 0.0149r^2(1-r^2)$ |
| 3b | 1.5 | 0.25 | 0.9 | 10 | 0.65 | 2 | $0.3282 – 0.1282r^2 – 0.0120r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.95 | 2 | $0.3221 – 0.1221r^2 – 0.0091r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.3996 – 0.1496r^2 – 0.0289r^2(1-r^2)$ |
| 4b | 1.5 | 0.35 | 0.9 | 10 | 0.35 | 2 | $0.4919 – 0.1419r^2 – 0.0253r^2(1-r^2)$ |
| | 1.5 | 0.45 | 0.9 | 10 | 0.35 | 2 | $0.5838 – 0.1338r^2 – 0.0216r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.3281 – 0.1281r^2 – 0.0128r^2(1-r^2)$ |
| 5b | 1.5 | 0.25 | 0.9 | 20 | 0.35 | 2 | $0.3148 – 0.1148r^2 – 0.0191r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 30 | 0.35 | 2 | $0.3066 – 0.1066r^2 – 0.0215r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.3 | 10 | 0.35 | 2 | $0.2406 – 0.0406r^2 – 0.0058r^2(1-r^2)$ |
| 6b | 1.5 | 0.25 | 0.6 | 10 | 0.35 | 2 | $0.2746 – 0.0746r^2 – 0.0047r^2(1-r^2)$ |
| | 1.5 | 0.25 | 0.9 | 10 | 0.35 | 2 | $0.2998 – 0.0998r^2 – 0.0036r^2(1-r^2)$ |

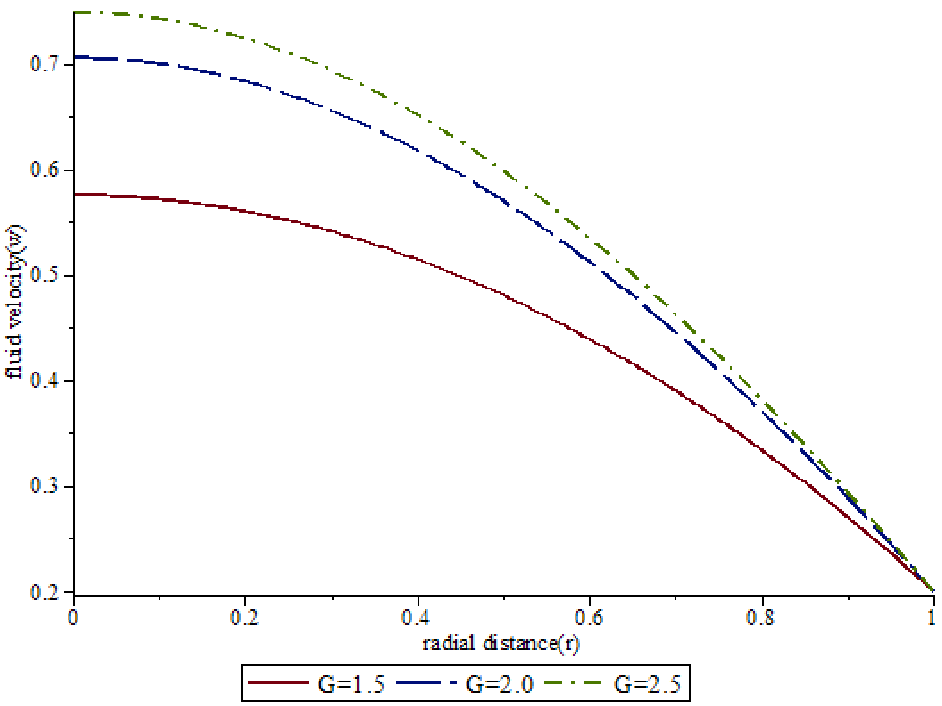


**Figure 2a: Variation of Velocity Profile of Blood along radial distance for different values of the Pressure Gradient for the blood flow with constant viscosity.**

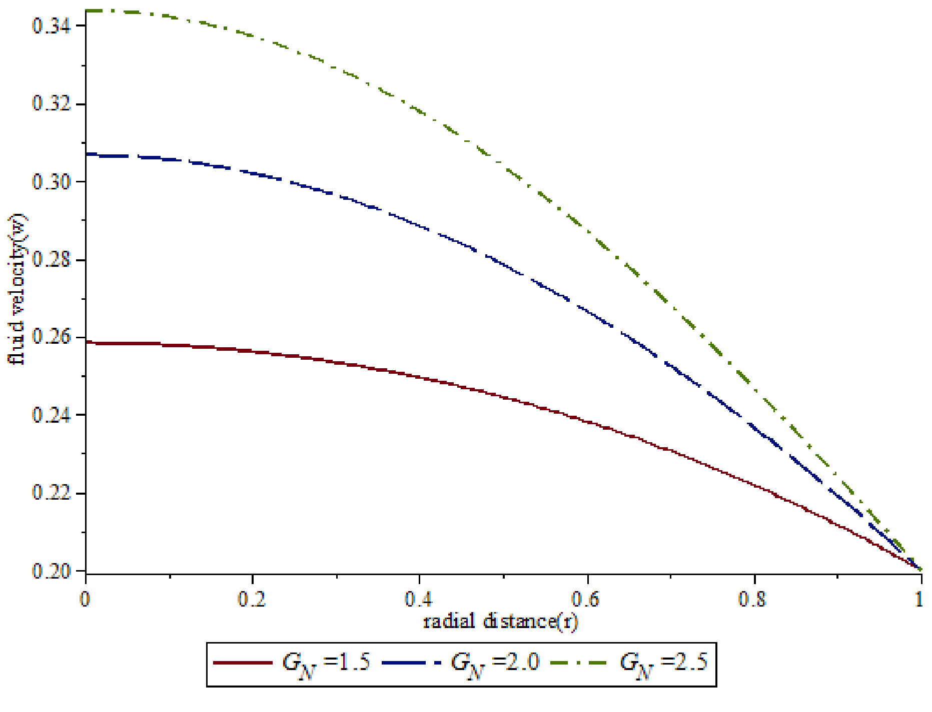


**Figure 2b: Variation of Velocity Profile of Blood along radial distance for different values of the Pressure Gradient for the blood flow with Variable Viscosity.**

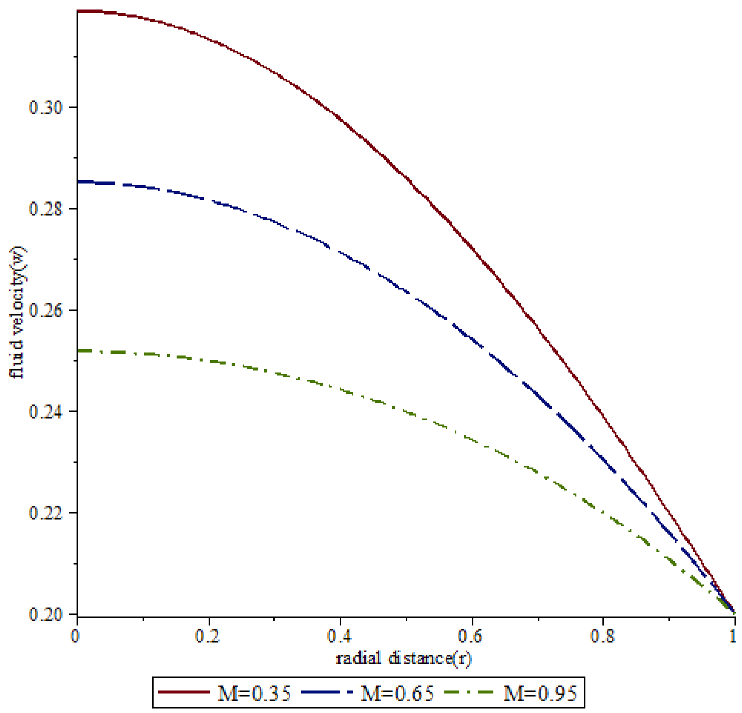


**Figure3a: Variation of Velocity Profile of Blood Flow for various values of Magnitude Field Parameter in the radial direction.**

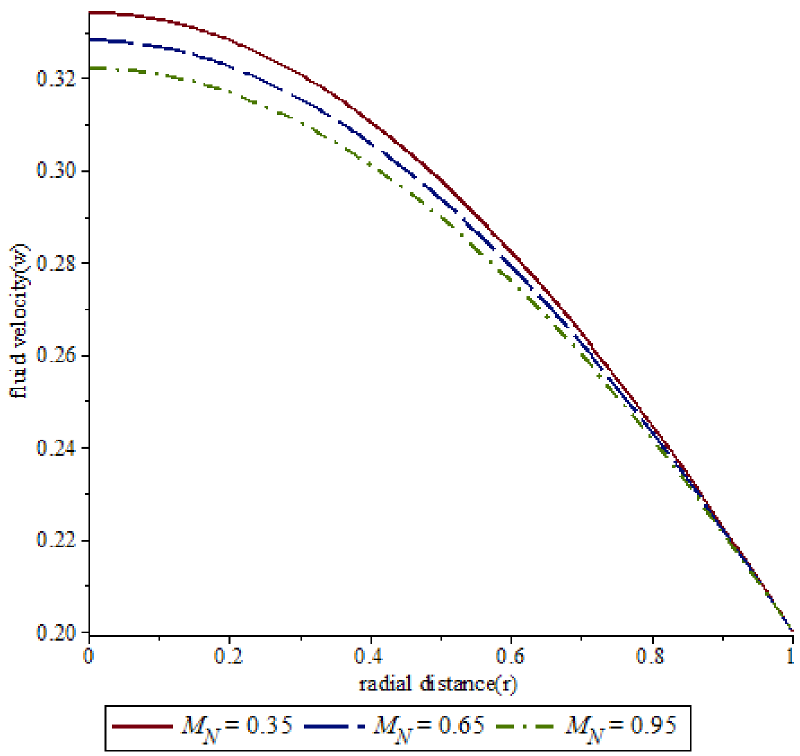


**Figure 3b: Variation of Velocity Profile of Blood along radial distance for different values of the Magnetic Field Parameter for the blood flow with Variable Viscosity.**

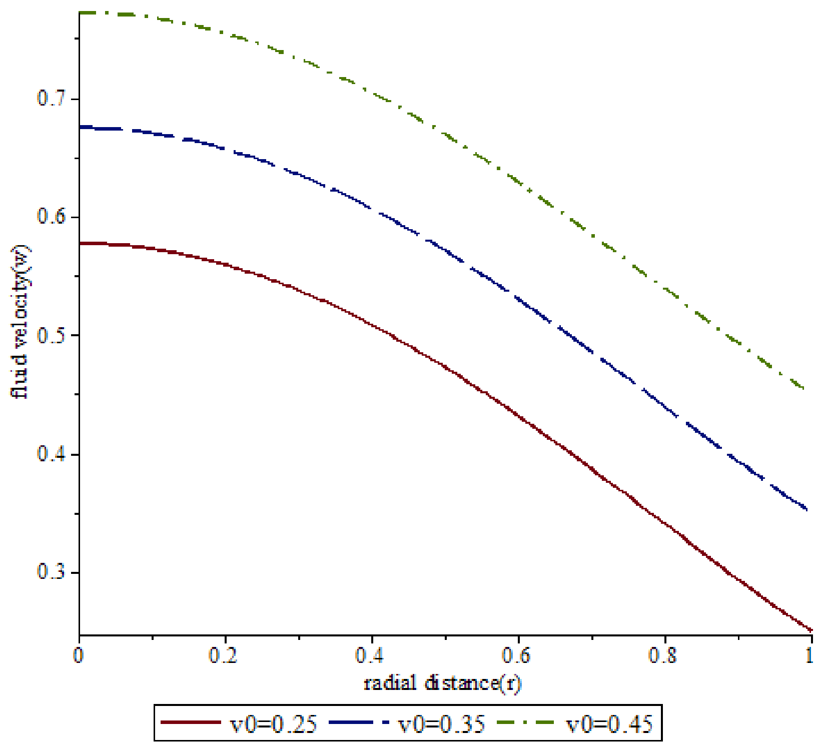


**Figure 4a: Variation of Velocity Profile of Blood along radial distance for different values of the Slip Velocity for the blood flow with Constant Viscosity.**

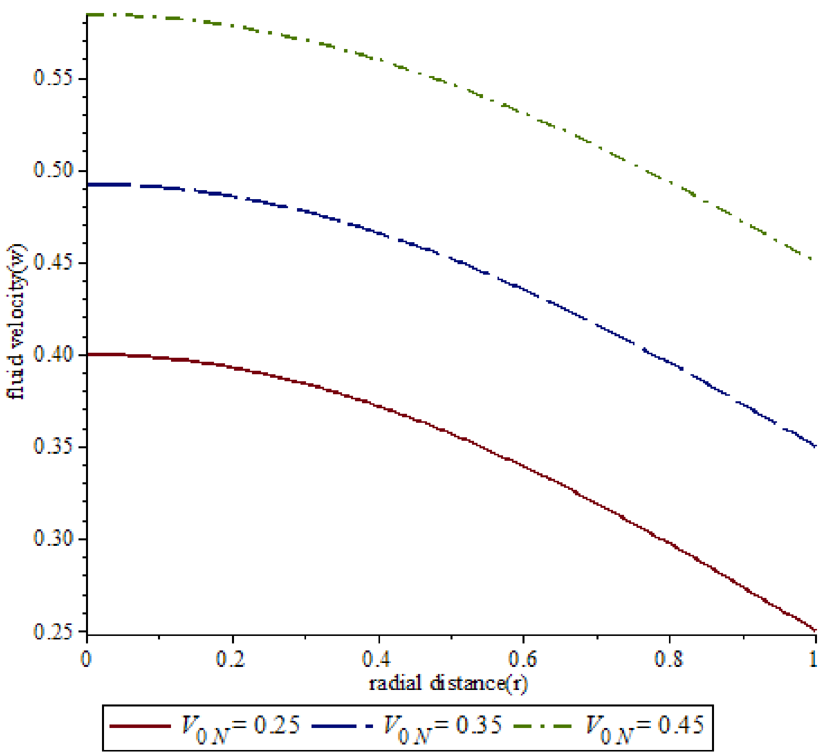


**Figure 4b: Variation of Velocity Profile of Blood along radial distance for different values of the Slip Velocity for the blood flow with Variable Viscosity.**

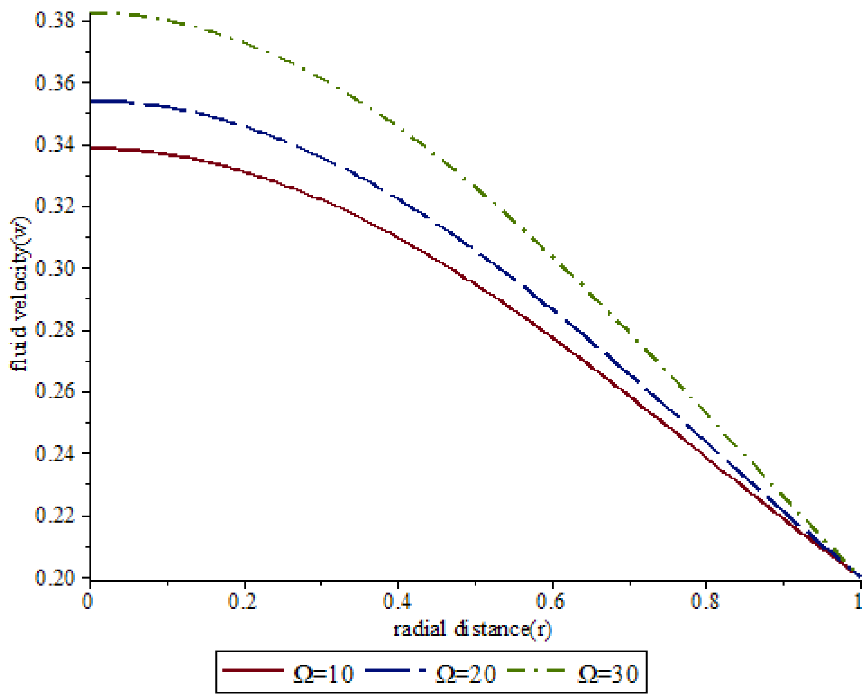


**Figure 5a: Variation of Velocity Profile of Blood along radial distance for different values of the Shear Thinning for the blood flow with Constant Viscosity.**

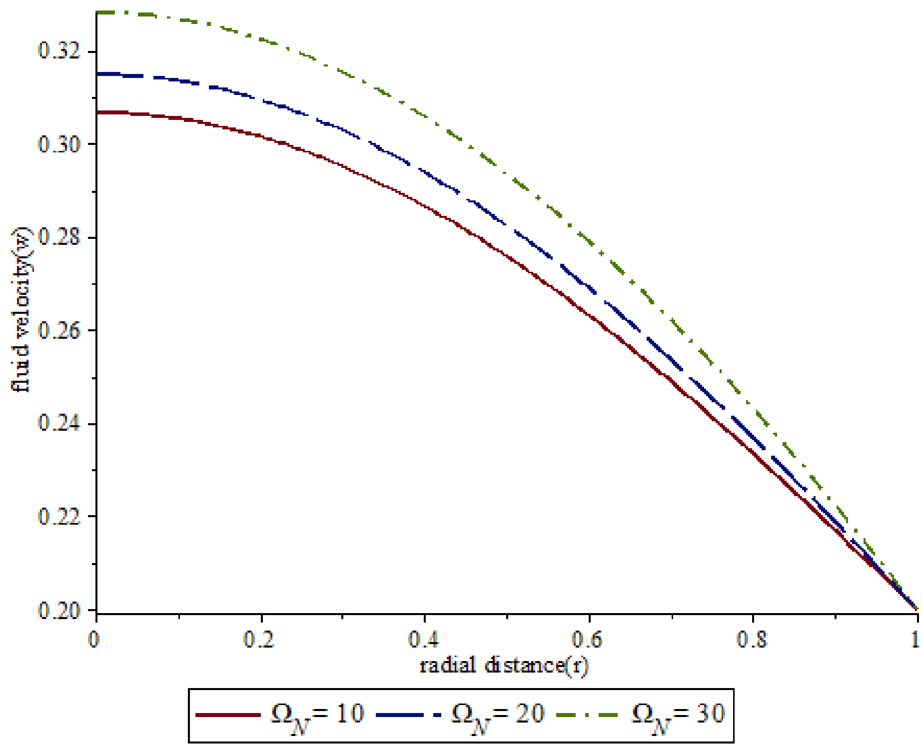


**Figure 5b: Variation of Velocity Profile of Blood along radial distance for different values of the Shear Thinning for the blood flow with Variable Viscosity.**

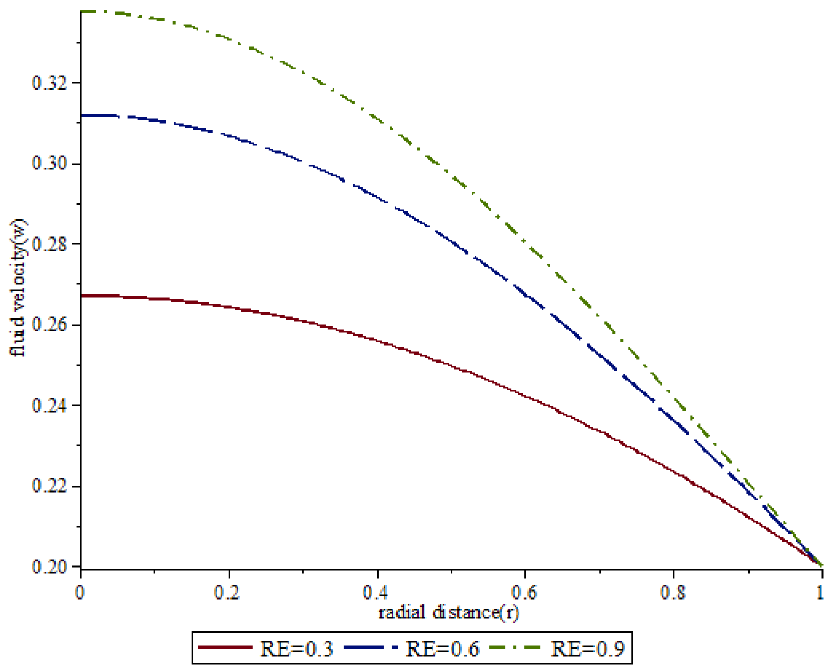


**Figure 6a: Variation of Velocity Profile of Blood along radial distance for different values of the Reynold number for the blood flow with Constant Viscosity.**

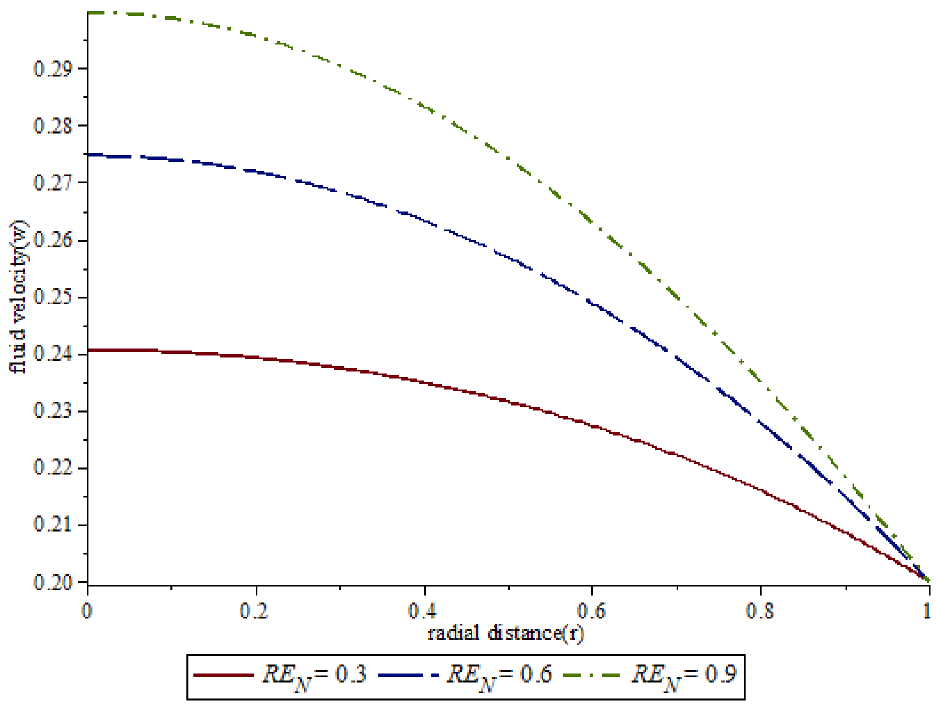


**Figure 6b: Variation of Velocity Profile of Blood along radial distance for different values of the Reynold number for the blood flow with Variable Viscosity.**

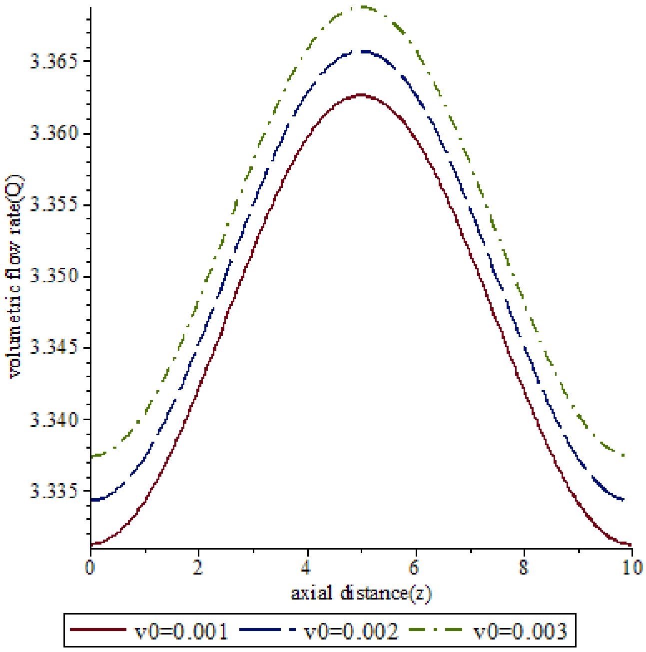


**Figure 7a: Variation of Volumetric Flow Rate of Blood Flow with constant viscosity for the increasing values of the Slip Velocity in the entire arterial region along the axial direction.**

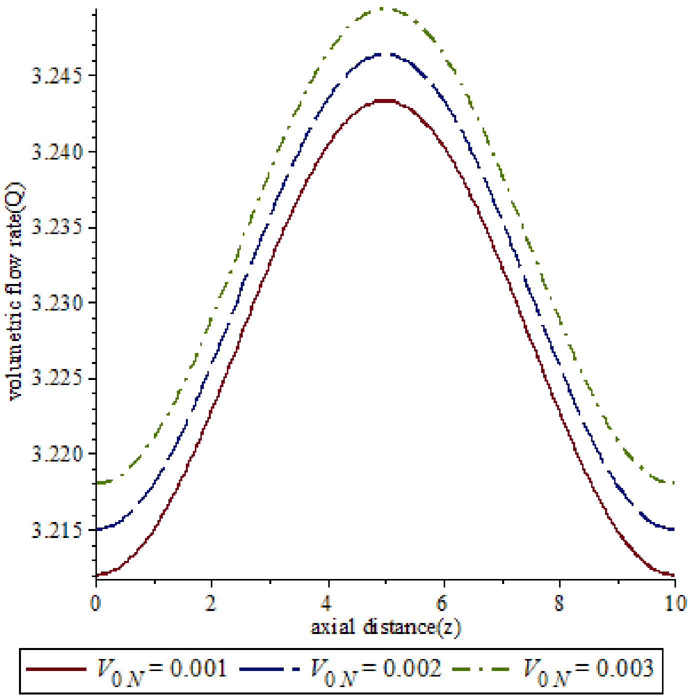


**Figure 7b: Variation of Volumetric Flow Rate of Blood Flow with Variable Viscosity for the increasing values of the Slip Velocity in the entire arterial region along the axial direction.**

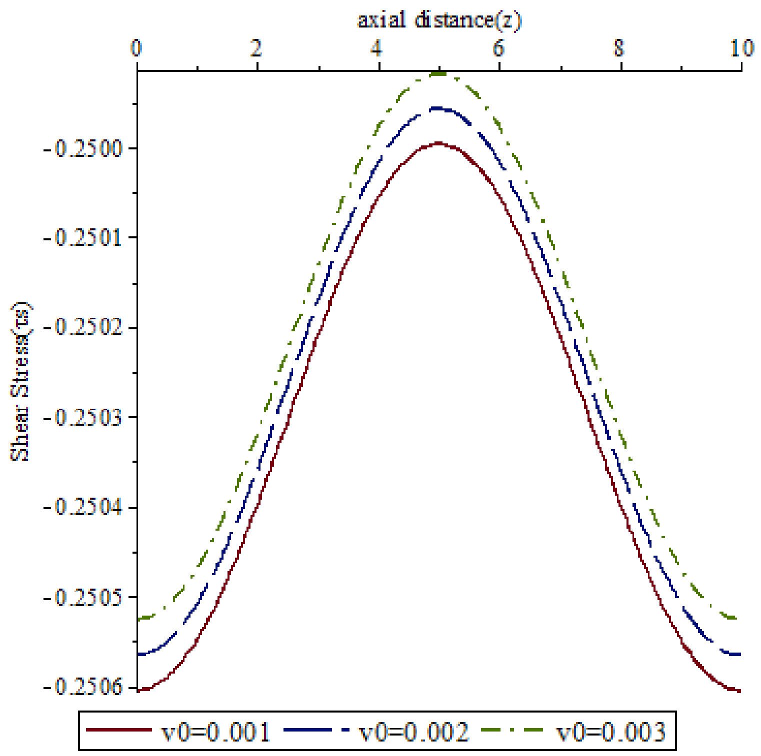


**Figure 8a: Variation of Shear Stress of Blood Flow with Constant Viscosity for the increasing values of the Slip Velocity in the entire arterial region along the axial direction.**

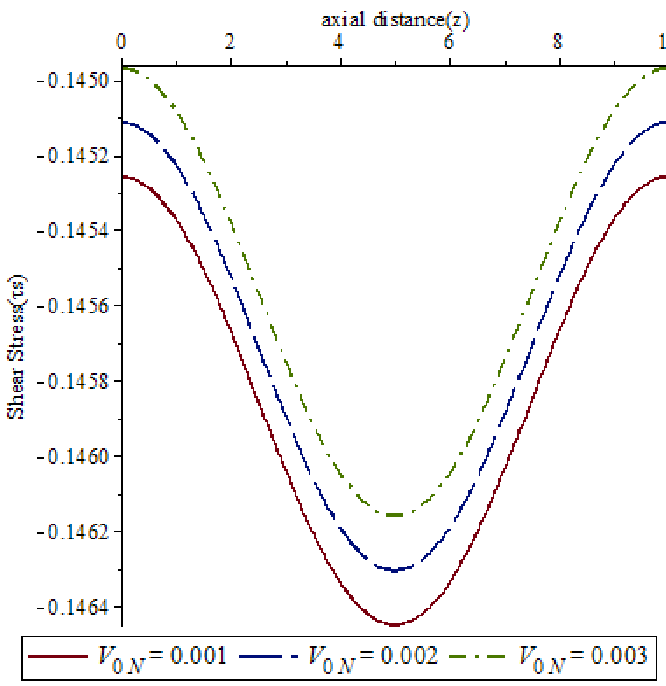


**Figure 8b: Variation of Shear Stress of Blood Flow with Variable Viscosity for the increasing values of the Slip Velocity in the entire arterial region along the axial direction.**

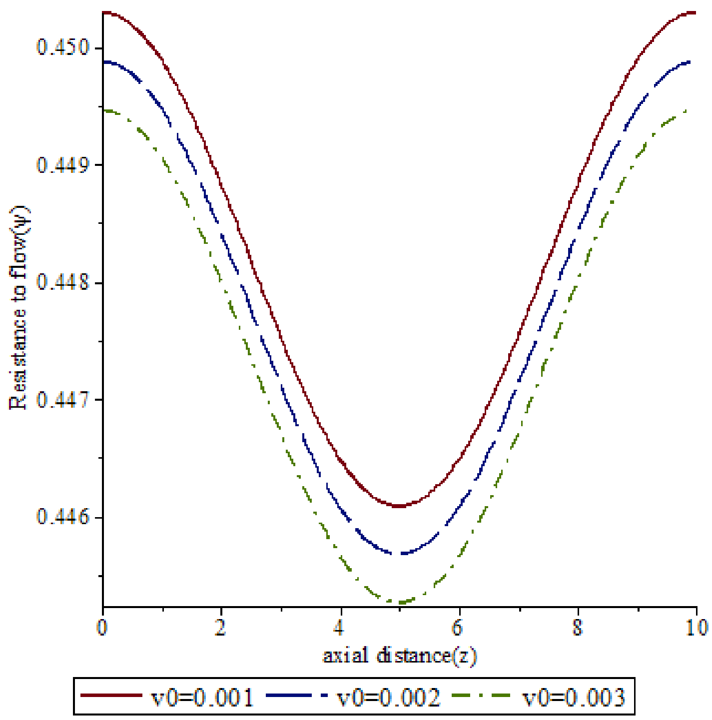


**Figure 9a: Variation of Resistance to Blood Flow with Constant Viscosity for the increasing values of the Slip Velocity in the entire arterial region along the axial direction.**

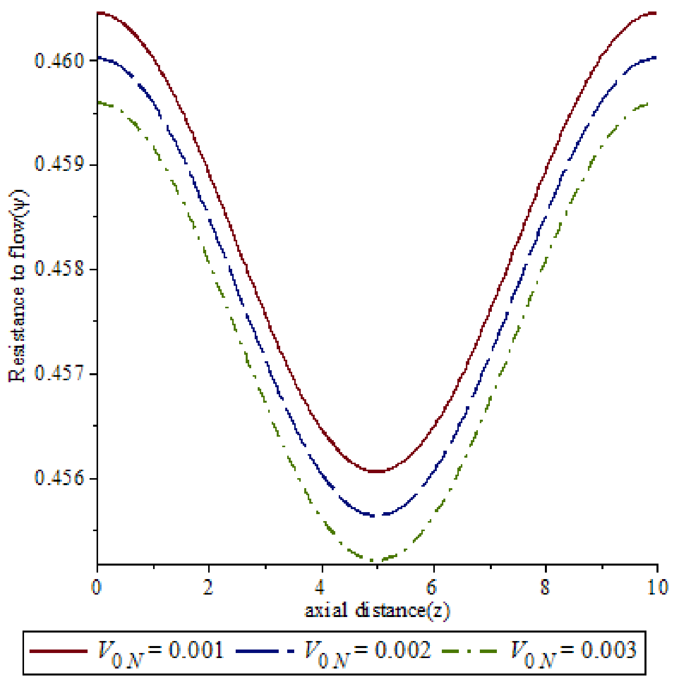


**Figure 9b: Variation of Resistance to Blood Flow with Variable Viscosity for the increasing values of the Slip Velocity in the entire arterial region along the axial direction.**

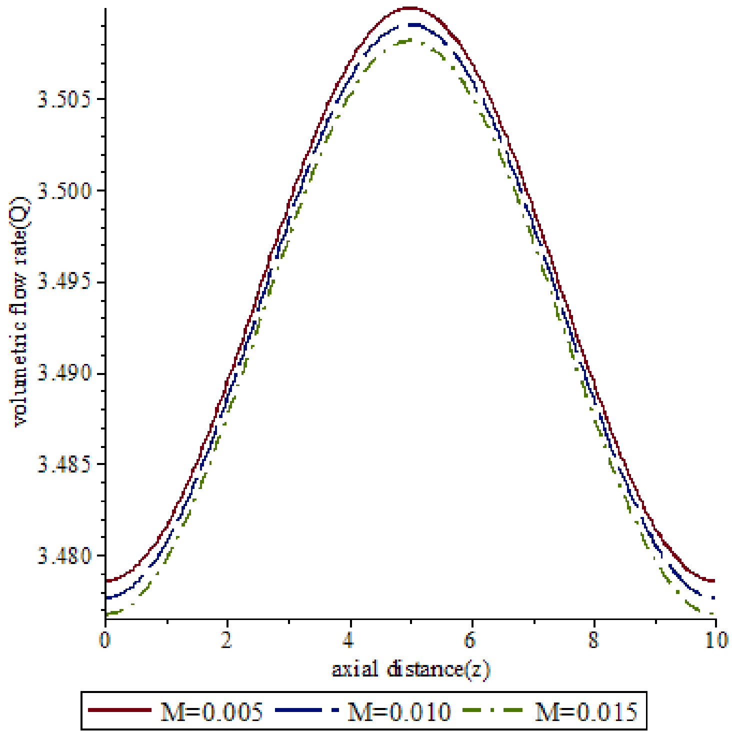


**Figure 10a: Variation of Volumetric Flow Rate of Blood Flow with Constant Viscosity for the increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.**

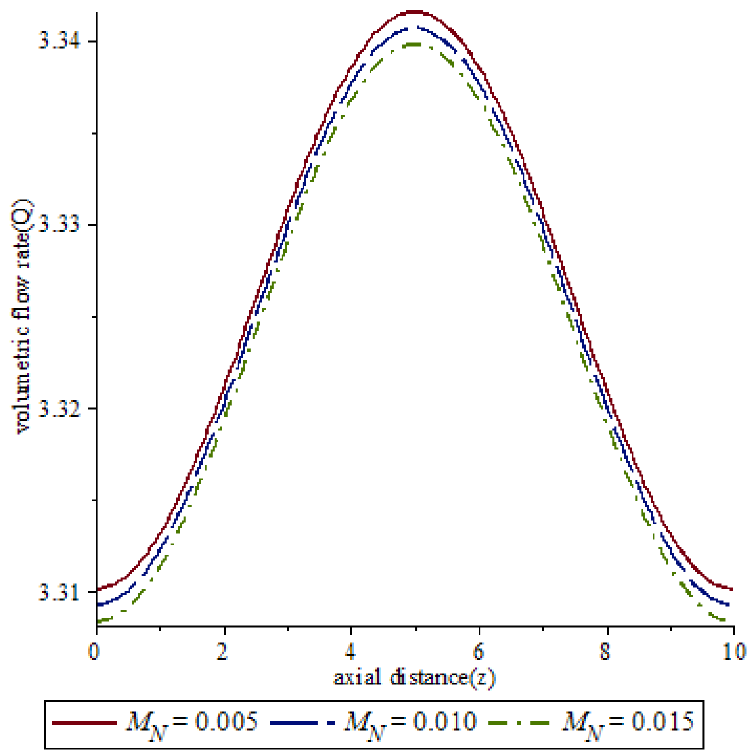


**Figure 10b: Variation of Volumetric Flow Rate of Blood Flow with Variable Viscosity for the increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.**

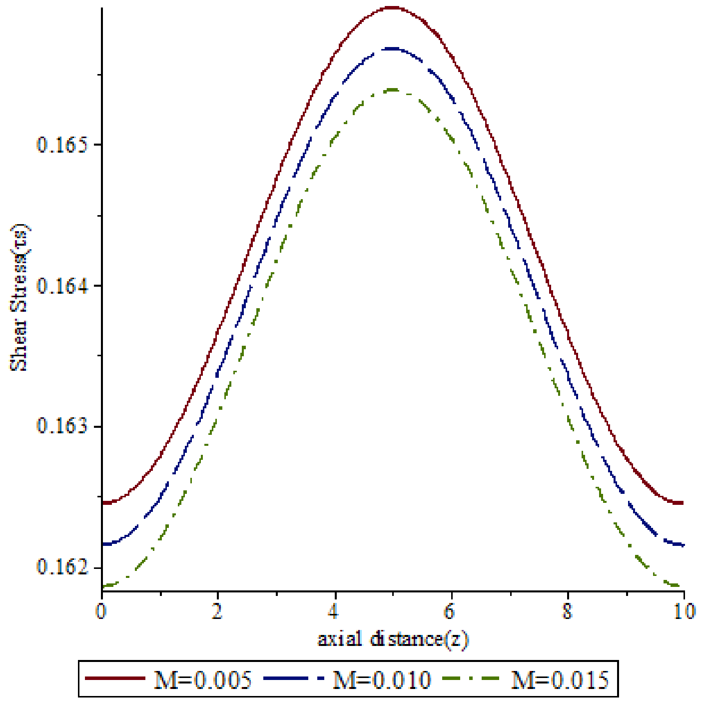


**Figure 11a: Variation of Shear Stress of Blood Flow with Constant Viscosity increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.**

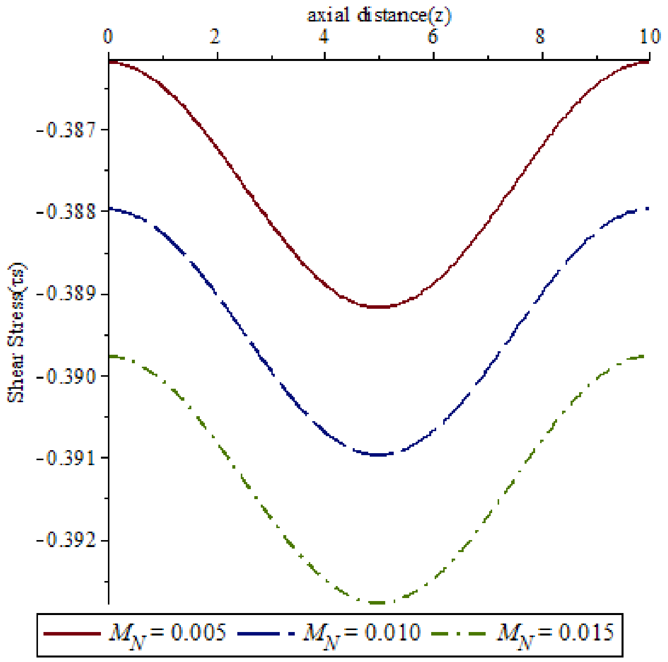


**Figure 11b: Variation of Shear Stress of Blood Flow with Variable Viscosity increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.**

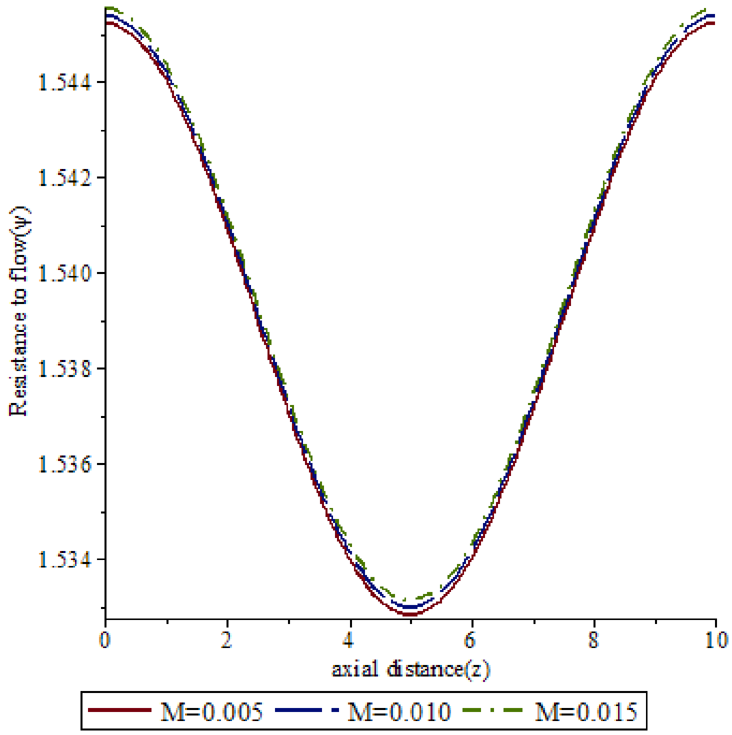


**Figure 12a: Variation of Resistance to Blood Flow with Constant Viscosity increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.**

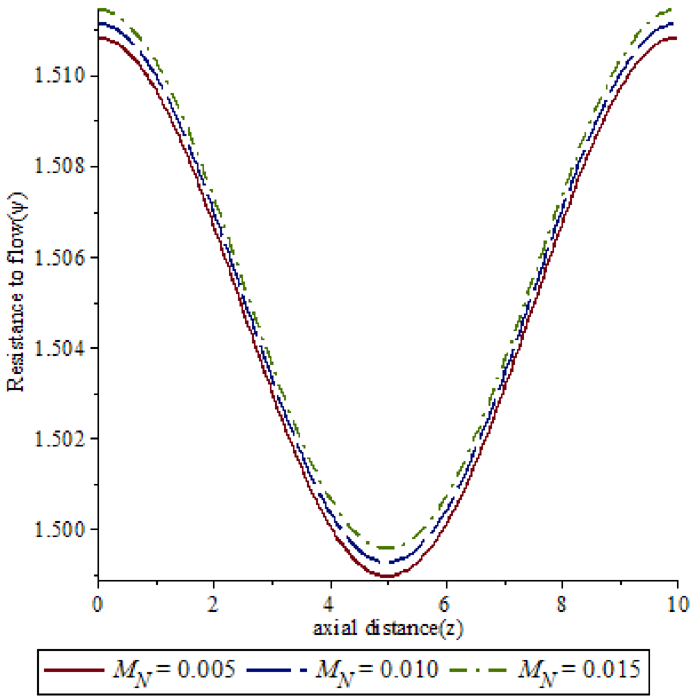


**Figure 12b: Variation of Resistance to Blood Flow with Variable Viscosity increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.**

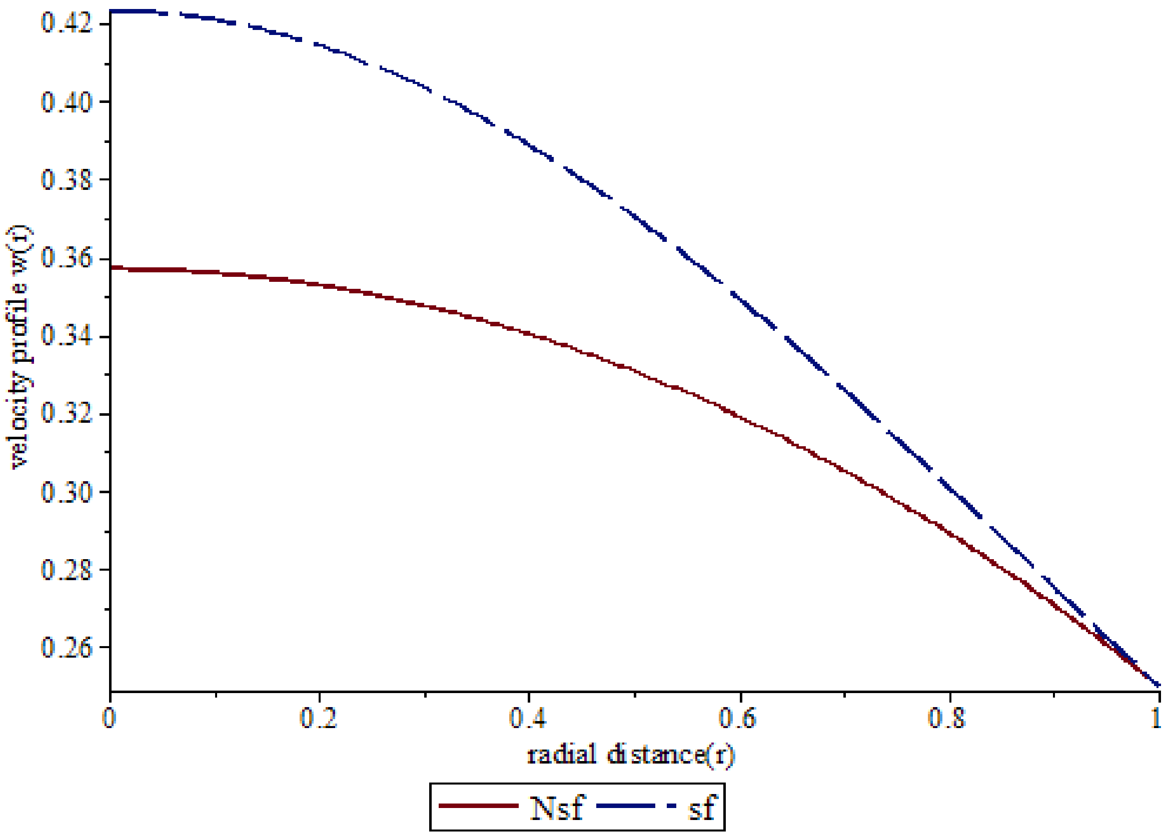


**Figure 13: Comparison of the Velocity Profiles of the Steady Blood Flow Model with Constant and Variable Viscosities in the Radial Direction.**

**4.0 Discussion of Results**

To illustrate the blood flow behavior with constant viscosity and variable viscosity dependent on red blood cells concentration (hematocrit), the results are shown graphically with the help of maple computer software. The effects of various parameters on flow velocity, volumetric flow rate, shear stress and resistance to flow for the blood flow models with constant and variable viscosities are calculated and shown graphically above.

Figures 2a and 2b show the variation of velocity profiles of blood flow models with constant and with variable viscosities for different values of the pressure gradient. It is reveals from the figures that pressure gradient increases with flow velocity but the effect of pressure gradient on the flow of blood with variable viscosity is more noticeable than constant viscosity. Figures 3a and 3b illustrate the effects of magnetic field on the flow velocity for the blood flow with constant and variable viscosities. It is found that magnetic field gradually decreases the flow velocity in both cases but the effect of the magnetic field is more noticeable on the blood flow model with constant viscosity than variable viscosity. Figures 4a and 4b reveal the variation of flow velocity along the radial distance as slip velocity increases for the blood flow models with constant and variable viscosities. The figures show that increase in slip velocity significantly lead to an increase in velocity profile. Figures 5a, 5b, 6a, and 6b shows the variation of velocity profiles for different values of shear thinning and Reynold number for the blood flow models with constant and variable viscosities. The figures reveal that, shear thinning and Reynold number increases with velocity profiles.

Also, figures 7a, 7b, 8a, 8b, 9a and 9b shows the variation of volumetric flow rate, shear stress and flow resistance for different values of the slip velocity for the blood flow models with constant and variable viscosities. It is reveals from those figures that, slip velocity increases with volumetric flow rate and shear stress but decrease the resistance to flow. It is also reveal that smaller value of slip velocity is required for more noticeable effects on the volumetric flow rate and shear stress for the flow model with variable viscosity compare to that of variable

Viscosity. In the same vein, small value of the slip velocity is required for more noticeable effect on the resistance to flow for the blood flow model with constant viscosity compare to that of with variable viscosity.

Figures 10a, 10b, 11a, 11b, 12a and 12b shows the variation of the volume flow rate, shear stress and resistance to blood flow along the axial direction for different values of the magnetic field for the blood flow models with constant and variable viscosities. The figures revealed that increase in magnetic field lead to increase in resistance to flow and decreases the volume flow rate and shear stress. The variation of the shear stress is in the positive direction for the blood flow model with constant viscosity while that of the variable viscosity is in the negative direction as indicated in figures 11a and 11b.

Finally, for the fixed values of all the parameters, the velocity profile of the blood flow with constant viscosity is higher than with variable viscosity as shown in figure 13.

## 5 Conclusion

Present study brings out many interesting results on rheological properties of blood flow through stenosed artery for the models with constant viscosity and variable viscosity of blood dependent on red blood cells concentration (hematocrit) considering blood as third fluid model. Since high blood viscosity is very dangerous for the cardiovascular disorders, the present models may be used as a tool for reducing the blood viscosity by using slip velocity at the constricted artery. Since the externally applied magnetic field gradually reduces the flow velocity and flow rate, the present study is useful for the reduction of blood flow during surgery and magnetic resonance imaging. In order to control blood pressure and blood viscosity, it is suggested to vary viscosity of blood with red blood cell concentration (hematocrit).

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**Nomenclatures**

w - Fluid velocity

$\bar{w}$ - Dimensionless fluid velocity

t - Time component

$\bar{t}$ - Dimensionless time component

r - Radial distance

y - Dimensionless radial distance

z - Axial distance

$w_s$ - Slip velocity

$V_0$ - Dimensionless Slip velocity

$\psi$ - Resistance to flow

$\upsilon$ - Dynamic viscosity

$R_0$ - Radius of the normal artery

R(z) - Radius of the artery in a stenotic region

$\beta_0$ - Magnetic Field Strength

Q - Volumetric flow rate

$\tau_s$ - Wall Shear Stress

Ƹ - Maximum height of the stenosis

L - Length of the stenosis

$RE$ - Reynold number for the flow with constant viscosity

$RE_N$ – Reynold number for the flow with variable viscosity

$M$- Magnetic field parameter for the flow with constant viscosity

$M_N$ - Magnetic field parameter for the flow with variable viscosity

$V_0$ - Slip velocity for the flow with constant viscosity

$V_N$ – Slip velocity for the flow with variable viscosity

$G$- Pressure gradient for the flow with constant viscosity

$G_N$ – Pressure gradient for the flow with variable viscosity

Ω- Shear thinning for the flow with constant viscosity

$\Omega_N$ - Shear thinning for the flow with variable viscosity

$sf$- Steady flow model with constant viscosity

$Nsf$- Steady flow model with variable viscosity