New strategy in modeling and prediction of GPS satellite orbits

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Abstract
The Operational Control Segment (OCS) of Global Positioning System (GPS) produce predicted navigation message which it is contain satellite ephemerides and clock corrections and utilized by receivers to calculate real-time satellite position and clock corrections for use in navigation solutions. The GPS based positioning and user accuracy are reduced in case of any errors in these ephemerides. In spite of broadcast ephemerides give satisfactory accuracy in numerous applications, they may not be sufficient for applications requiring high accuracy and it is needful to model the satellite orbit. The GPS based positioning process requires appropriate observation and prediction and computation of precise satellite orbits with adjustment techniques. Because the release period of IGS products going from around 1 day to about fourteen days which is a major disadvantage for high precision GNSS processing, this paper was done. In this paper a comparison was carried out between the satellite position calculated utilizing precise position provided by the Final Orbit solution from International GPS Service for Geodynamics (IGS), and the history broadcast ephemerides data in order to modeling and prediction of GPS satellite orbits from navigation file. The proposed method does not involve complex dynamic models, but takes full advantage of relatively mature observable history data of the orbit. The new satellite orbit prediction method is based on Fourier series and used to predict satellite orbit at some time in the future by using broadcast ephemeris history data and the satellite's position error limited to 35 cm for the first 12 hours and this errors in predicted orbit increased to sub-meter for the second 12 hours and this errors are increased to be bigger than 5 m after 60 hours.

Key words: Orbit determination; broadcast ephemeris; precise ephemeris.

1. Introduction

The NAVigational System utilizing Timing And Ranging (NAVSTAR) Global Positioning System (GPS) Operational Control Segment (OCS) creates predicted navigation message which it is contain satellite ephemerides and clock corrections and utilized by receivers to calculate real-time satellite position and clock corrections for use in navigation solutions. The generation of the navigation message generation begins with the OCS.'s utilization of a Kalman filter to calculate satellite position, velocity, clock bias, clock drift, clock drift rate; and coefficients of solar radiation pressure. These evaluated parameters are then utilized to calculate the satellite position and clock corrections into the future. The calculated values are then fit to a set of equations and the fit coefficients are broadcast in the navigation message. **[1]**

It is consider that the coordinate calculation of satellites inside its orbit is an important step of determining GNSS receivers position, both during post-processing of raw measurement data, and in real time. Exact orbital parameters are given in 13-day delay. It is worth mentioning that the accuracy of calculating the satellite position inside its orbit by the GNSS receiver in real time is greater when it is compared with calculation using the almanac data. **[2]** The estimated orbit for each satellite utilizing the broadcast ephemerides is vary from their actual satellite positions as appeared in Figure 1. This variation is because of distrust in the gravitational model, insufficiently accuracy of the orbit exemplifications and deficiently modeled surface forces on the satellites like atmospheric drag and solar radiation pressure. In this paper, the orbital solution errors are estimated by comparing and find the difference between the broadcast ephemerides and the precise ephemerides. **[3]**

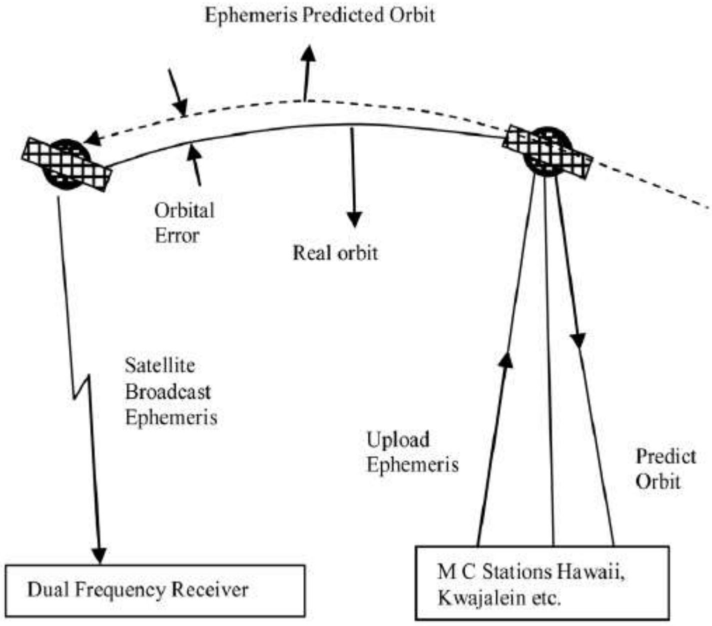

Figure 1. Satellite ephemeris errors. **[3]**

There are basically two diverse orbital information, in particular broadcast ephemerides and IGS final ephemerides (IGS rapid, ultra rapid, predicted and final ephemerides) utilized in the GPS positioning. The broadcast ephemerides utilized practically and in real time are acquired through evaluations got from the USA GPS reference stations observations. Contingent upon GPS week, Broadcast ephemerides are created from satellite data and the accuracies they give are sufficient in numerous GPS applications. Then again, a lot of parameters (for instance, data about gravity region, enhanced satellite orbit data, etc.) should be known so as to obtain high accuracy in geodetic and engineering applications. Final ephemeris data can be downloaded from the related sites by means of the web. In this paper, Keplerian motion and Keplerian orbital parameters will be clarified quickly and a broad information about ephemerides will be given. Inside this extension, satellites ECEF coordinates were computed using the broadcast ephemerides and this coordinates were compared with the coordinates obtained from the final orbits of IGS. **[4]** The paper is organized as follows. Section 2 discusses orbit determination. While experimental analysis and results is presented in Sections 3, conclusions are given in Section 4.

2. Orbit determination

There are fundamentally two various orbital data, specifically broadcast ephemerides and IGS final ephemerides (IGS rapid, ultra rapid, predicted and final ephemerides) utilized in the GPS positioning. The broadcast ephemerides utilized practically and in real time are acquired through evaluations got from the USA GPS reference stations observations. **[4]**

2.1 Broadcast ephemerids

Ephemeris data is the data utilized to GPS calculation of satellite position in ECEF coordinate system. This data is propagated by every individual satellite and valid and suitable for that satellite only. **[5]**

The Kalman filter is utilized by OCS's to calculate satellite position, velocity, clock bias, clock drift and clock drift rate, solar radiation pressure coefficients from navigation message.

Using these evaluated parameters, the satellite position and clock corrections can be calculated in the future. After that a fitting processes is performed to the values of propagated parameters to a set of equations and the fit coefficients are circulate in the navigation message as broadcast ephemerides. **[6]** To perform any navigation tasks, the real time satellite positions and satellite system time must be known. The data signal which includes orbital information is broadcast by means of navigation message. Control Unit determines navigation message and transmits as “broadcast” to users via GPS satellites. The broadcast ephemeris in the navigation message file contains Keplerian parameters which are used to calculate the coordinates and clock correction for each satellite. The following items are calculated at epoch by using broadcast;

− Satellite position.

− Satellite velocity.

Figure 2 shows the form of perturbation parameters and Keplerian elements which are used to calculate the anticipated satellite positions In process of Kalman Filter. Table 3 shows and summarizes all parameters defining the satellite orbit and the state of the satellite clock. Depends on a curve fit for a period of four hours, t_{0e} and t_{0c} are the parameters which are refer to a given reference epoch for the ephemeris and the clock respectively. Subsequently, the representation of the satellite route is accomplished through a series of several bothered Keplerian orbits. Table 3 shows the groups of the parameter which are utilized to calculate satellite coordinates and satellite time. The first set of parameters are concerned of real satellite time while the Kepler ellipse at reference epoch is defined by the second set whereas the nine perturbation parameters is included in third set. **[4]**

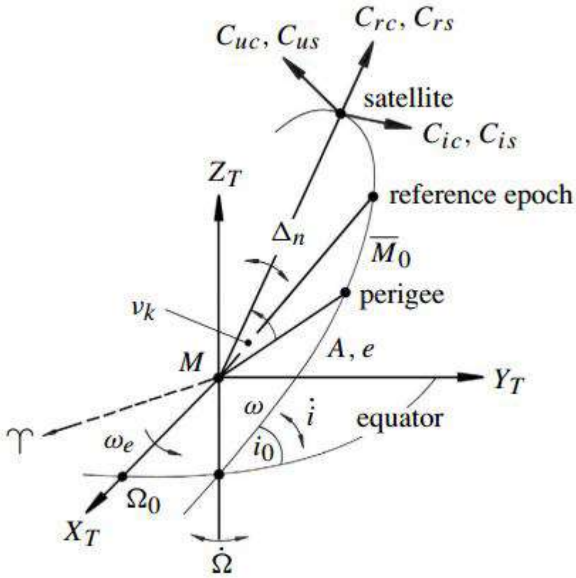

Figure 2 : Keplerian and perturbed parameters in broadcast ephemerides **[4]**

Table 3 : Parameters of Broadcast Ephemerides

Time Parameters	
t_{0e}	Reference time, ephemerides parameters [s]
t_{0c}	Reference time, clock parameters [s]
a_0, a_1, a_2	Polynomial coefficients for clock corrections (bias [s], drift [s/s], drift rate [s/s^2])
IODC	Issue of Data, Clock, arbitrary identification number
Keplerian Parameters	
$\sqrt{\mathbf{a}}$, e, i_0, Ω_0, ω, M_0	Keplerian elements of T_{0e}
IODE	Issue of Data, Ephemeris, arbitrary identification number
Perturbation Parameters	
Δn	Mean motion difference from computed value [semicircles/s]
di/dt (or IDOT)	Rate of change for inclination angle, (radian/second)
Ω	Rate of change in ascending node right ascension
Cuc, Cus	Correction coefficients for perigee argument, (radian)
Crc, Crs	Correction coefficients for geocentric distance, (meter)
Cic, Cis	Correction coefficients for inclination angle, (radian)

For more details with parameters symbols, the parameters, $\sqrt{A}, e.i_o, \omega, \Omega_o, M_o$ determine a standard ellipse orbit, the deviation between the actual satellite orbital motion and standard orbit is defined by the six sine-cosine parameters $C_{ic}, C_{is}, C_{rc}, C_{rs}, C_{us}, C_{uc}$. The GPS satellites broadcast ephemeris is broadcasted once at regular intervals every two hours, at any time the coordinate of satellite orbit would be determined with the parameters $\Delta n, \dot{\Omega}, i$ relation to the reference time, which also called broadcast time. **[7]** At a specific time, the ECEF coordinate of GPS satellite can be calculated by utilizing the broadcast ephemeris parameters in RINEX (Receiver Independent Exchange format) navigation file. **[4]** Table 4 shows an example of part of this file at certain time for satellite vehicle # 1, and table 5 shows corresponding parameters symbol and position in navigation file. From these parameters one is able to compute a highly accurate position of the satellite in ECEF coordinates under normal conditions. **[5]**

Table 4 : Example of part of RINEX navigation data file for satellite PRN # 1

2	17 1 1 2 0 0	.501201022416D-03	-.636646291241D-11	.000000000000D+00
	.380000000000D+02	-.111062500000D+03	.467840916023D-08	-.130855169793D+01
	-.561773777008D-05	.163936362369D-01	.921264290810D-05	.515370865631D+04
	.720000000000D+04	.381842255592D-06	.664509310566D-01	.316649675369D-06
	.945435967992D+00	.192218750000D+03	-.204796804639D+01	-.809676583423D-08
	-.176078762959D-09	.100000000000D+01	.193000000000D+04	.000000000000D+00
	.240000000000D+01	.000000000000D+00	-.200234353542D-07	.380000000000D+02
	.600000000000D+01	.400000000000D+01		

Table 5 : RINEX navigation data file parameters symbol

PRN	YY MM DD H M S	a0	a1	a2
	IODE	C_{rs}	Δn	M_0
	C_{uc}	e	C_{us}	$\sqrt{A}$
	TOE	C_{ic}	Ω_0	C_{is}
	i_0	C_{rc}	ω	$\dot{\Omega}$
	IDOT	Codes_on_L2_ch	GPS_Week_#	L2_P_data_flag
	SV_accuracy	SV_health	TGD	IODC
	Trans_Time	Fit_interval	Spare	Spare

2.2 Precise orbit

Globally distributed tracking stations play an important role in determining a posteriori precise ephemerides (PE) and clock parameters. The code phases and carrier phases of all satellites in view can be measure by using a dual-frequency receivers installed at such stations. The format of data files more often adjust to the SP3 data format (Standard Product 3), settled by the U.S. National Geodetic Survey (NGS). The precise of this format getting between 1 mm and 1 picosecond. A few organizations give precise ephemerides and the parameters of adjusted clock, for example the NIMA, JPL, and IGS. The position and velocity vectors for each satellite are given at regular intervals every 15 minutes. In GPS post processing data the precise orbits and adjusted clock parameters can be used, similarly the precise orbits used in Precise Point Positioning (PPP) mode to processing of single receiver data. The NIMA precise ephemerides are accessible through Internet freely. Another resource of precise ephemerides and adjusted clock parameters for precise orbits are the NASA JPL (Jet Propulsion Laboratory). The regular intervals are given at every 15 minutes for positions and velocities and every 5 minutes for clock parameters. Final orbits are accessible after around about fourteen days. Rapid orbits are allowed inside 20 hours; they concur with the precise of final orbits at the level of around 20 cm. The IGS, a service established by the IAG, is considered as the most important source for GPS products and precise ephemerides. After a fruitful pilot period of over one year, the IGS formally began its activities on January 1, 1994. The IGS name was became International GPS Service in 1999 after it was named International GPS Service for Geodynamics. Generally, the essential target of the IGS is to give a service to help through GPS products and GPS data, and the activities of geodetical and geophysical research. IGS have globally distributed stations over the world numbering more than 506 stations through these stations; IGS collects, archives and distributes GPS observation data. In all these stations a certain quality criteria have to be met. The weekly final products, which are the IGS core products, are;

− GPS ephemeris and values of clock, which is organized in table for each day at regular interval of 15-minutes (in SP3 format),
− Earth Orientation Parameters (EOP), and
− Geocentric station coordinates and velocities.

All necessities for precise orbits are totally satisfied with the nearby of IGS products. Side by side with the station of precise coordinates and the observation of original data from IGS stations, it became conceivable to connect directly every worldwide new station to the geocentric reference frame. **[8]**

The records of precise orbit from GPS week 649 (1992) to now are maintained by the website of IGS. **[6]** At the present, IGS is accountable for gathering, archiving and distribution of GPS measurements that could be used, with satisfactory accuracy, in engineering applications and scientific research. The measurements GPS are utilized to acquire the following products.

- Ephemerides of GPS with high accuracy.
- Earth rotation parameters (ERP)
- IGS monitor stations coordinates and velocities.
- Clock data having a place with GPS satellites and monitor stations of IGS.
- Tropospheric zenith path delay calculation.

IGS products empower enhancement and advancement of the ITRF system, determination of earth's crust movements, determination of sea surface changes and give high accuracy required by ionospheric studies. IGS plays out these tasks within the following structure.

- A global observation network consisting of 506 stations
- Three global data centers

* IGN (Institute Geographies National, France)
* CDDIS (Crustal Dynamics Data Information System at Goddard Space Flight Center, USA)
* SIO (Scripps Institution Oceanography)

- Seven centers of analysis; CODE, NRCAN (EMR), ESA, JPL, GFZ, NGS and SIO.

The analysis centers task is to create day by day worldwide data without interruption.
Four unique parts of orbital data are produced by IGS according to orbits and clocks: IGS-Ultra-Speed, IGS Speedy, IGS Result orbital information (Table 6). **[4]**

Table 6: IGS GPS satellite and clock accuracy

Type	Accuracy		Latency	Updates	Sample Interval
Broadcast	orbits	~100 cm	real time	--	daily
	Sat. Clocks	~5 ns RMS ~2.5 ns Sdev			
Ultra-Rapid (predicted half)	orbits	~5 cm	real time	at 03, 09, 15, 21 UTC	15 min
	Sat. Clocks	~3 ns RMS ~1.5 ns Sdev			
Ultra-Rapid (observed half)	orbits	~3 cm	3 - 9 hours	at 03, 09, 15, 21 UTC	15 min
	Sat. Clocks	~150 ps RMS ~50 ps Sdev			
Rapid	orbits	~2.5 cm	17 - 41 hours	at 17 UTC daily	15 min
	Sat. & Stn. Clocks	~75 ps RMS ~25 ps Sdev			5 min
Final	orbits	~2.5 cm	12 - 18 days	every Thursday	15 min
	Sat. & Stn. Clocks	~75 ps RMS ~20 ps Sdev			Sat.: 30s Stn.: 5 min

2.3 Calculation of ECEF Satellite Coordinates from Ephemeris Data

The satellite coordinates estimation from the broadcast ephemeris is done in two phases. At first, the satellite cartesian coordinates are estimated in the plane of orbit. Then a rotation is carried out through angle of inclination, in order to make the z axis coincident with the terrestrial z axis. At last, to make the x axis coincident with the Greenwich Meridian, a rotation of coordinates must be carried out by the angle Ω about the z axis. **[9]**

The time of GPS system is recognized by a week number and the number of seconds from the start of the present week; the GPS time can consequently differ between 0 toward the start of a week and 604800 toward the finish of a week. The epoch at January 5, 1980 at 0^h UTC is the first GPS epoch and the GPS week begins at midnight (Universal Time) between Saturday and Sunday. Subframe 1 of the navigation message contains the number of GPS week which is substituted by 10 bits. Henceforth the maximum value for the week number does not exceed 1023, and when the week number reaches 1023, the week number "rolls over" to zero. This event is called the End of Week (EOW) rollover. Therefore, there are courses of week numbers and the first cycle ends on August 21, 1999 and the second session lasts from August 22, 1999 and ends on April 6, 2019. The time of GPS system is a uninterrupted time scale, and is characterized by the weighted mean of the atomic clocks in the satellites and monitor stations. It is worth mentioning that there is no match in the GPS clocks mean drift between the time in the GPS system and UTC. This mismatch and difference is constantly defined by the control segment and sent to the user via navigation message. The oscillator of rubidium or cesium clocks for each satellite is observed by the control segment, and forecasted as a second degree polynomial. The first parameter group of table 3 resent these polynomial coefficients. The GPS system time, t is estimated from each satellite time, t_{SV} from this formula.

$$t = t_{SV} - \Delta t_{SV} \quad \ldots\ldots (1)$$

Where:

$$\Delta t_{SV} = a_0 + a_1(t - t_{0c}) + a_2(t - t_{0c})^2 \quad \ldots\ldots (2)$$

And t_{0c} is the reference epoch for the coefficients a_0, a_1, a_2. The GPS time is controlled to be within one microsecond of UTC modulo one second by OCS. But actually GPS time was retained to be within about 10 nanoseconds. The value of the term a_0 is about a few nanoseconds in the satellite message. In the further calculation and without sacrificing accuracy it can be substitute t_{SV} instead of the time parameter t. To get an expression for satellite clock drift, equation (2) must be differentiate with respect to time as:

$$\dot{\Delta} t_{SV} = a_1 + 2a_2(t - t_{0c}) \quad \ldots\ldots (3)$$

The coordinates X_K, Y_k, Z_k of the satellite are calculated for each epoch, t, as for to the Earth-fixed geocentric reference frame X_T, Y_T, Z_T. The elapsed time, t_k, since the reference epoch, t_{0e}, is

$$t_k = t - t_{0e} \quad \ldots\ldots (4)$$

A possible change of the week has to be considered. Two constants are required:

The WGS 84 value of the geocentric gravitational constant $GM = 3.986005 * 10^{14}\, m^3/s^2$, the WGS 84 value of the Earth rotation rate $\omega_e = 7.292115 * 10^{-5}\, rad/s$, and $\pi = 3.1415926535898$

Moreover we use:

$A = \left(\sqrt{A}\right)^2$ which is the semi-major axis

$n_0 = \sqrt{\frac{GM}{A^3}}$ which is the calculated mean motion

$n = n_0 + \Delta n$ which is the corrected mean motion

$M_k = M_0 + nt_k$ which is the mean anolaly

And the Kepler's equation of the eccentric anomaly
$E_k = M_k + e \sin E_k$ …………………………………………………………… (5)
Because of the GPS orbits are very small eccentricity this equation is solved by iteration. (e < 0.001) two steps are usually sufficient:
$E_0 = M, \; E_i = M + e \sin E_{i-1}, i = 1, 2, 3, \ldots$ ………………………………… (6)
By the following equations, the satellite coordinates can be obtained.
$\cos v_k = \frac{\cos E_k - e}{1 - e \cos E_k}$ which is the true anomaly …………………………………. (7)
$\sin v_k = \frac{\sqrt{1-e^2} \sin E_k}{1 - e \cos E_k}$ which is the true anomaly …………………………………. (8)
$\Phi_k = v_k + \omega$ which is argument of latitude ………………………………….. (9)
$\delta_{uk} = C_{uc} \cos 2\Phi_k + C_{us} \sin 2\Phi_k$ which is argument of latitude correction ….. (10)
$\delta_{rk} = C_{rc} \cos 2\Phi_k + C_{rs} \sin 2\Phi_k$ which is radius correction ………………….. (11)
$\delta_{ik} = C_{ic} \cos 2\Phi_k + C_{is} \sin 2\Phi_k$ which is inclination correction ……………….. (12)
$u_k = \Phi_k + \delta_{uk}$ which is corrected argument of latitude …………………….. (13)
$r_k = A(1 - e \cos E_k) + \delta_{rk}$ which is corrected radius ……………..………… (14)
$i_k = i_0 + \dot{i} t_k + \delta i_k$ which is corrected inclination ……………………………… (15)
$X'_k = r_k \cos u_k$ which is the position in the orbital plane……………………… (16)
$Y'_k = r_k \sin u_k$ which is the position in the orbital plane……….………………… (17)
$\Omega_k = \Omega_0 + (\dot{\Omega} - \omega_e) t_k - \omega_e t_{0e}$ is the corrected longitude of ascending node. ..(18)
$X_k = X'_k \cos \Omega_k - Y'_k \sin \Omega_k \cos i_k$ the earth fixed geocentric satellite coordinates.(19)
$Y_k = X'_k \sin \Omega_k + Y'_k \cos \Omega_k \cos i_k$ the earth fixed geocentric satellite coordinates.(20)
$Z_k = Y'_k \sin i_k$ which is the earth fixed geocentric satellite coordinates………... (21)
[8]

3. Experimental analysis and Results

The form of required data for this calculation is a broadcast and a precise ephemeris. The broadcast ephemeris is accessible from the GPS, as a set of parameters is sent to the user via the navigation message. The control center frequently update parameters every 2 hours and the computed coordinate accuracy is about 3 m. Different agencies all over the world produce many types of precise ephemerides by using and assisting the permanent GPS sites data. The accuracy of precise ephemerides ranges from 0.05 m for the post-processed orbit to 0.2 m for predicted orbit. **[10]** The distributed frequently format of precise ephemerides are SP3, and the regular intervals of coordinates and errors in satellite clock is 15-minunts for all GPS satellites. The broadcast orbits were determined from the broadcast ephemeris using the method described in the previous section. The user can be daily uploaded ephemeris data and this data are proper for a period 2 h. To match with the epochs of the IGS orbit each satellite coordinates was calculated at regular intervals 15-minuts utilizing the broadcast ephemeris. The coordinates of the first source of the SP3 files and the second source of the coordinates of the navigation files were calculated for a period of 60 hours. Figure 2, 3, and 4 represent his relationship between the time in hours as horizontal axis and the values of the ECEF for the satellite PRN#1 as the y axis in kms unit for the coordinates in the three directions X, Y and Z. In these three figures, its clear that there is a congruent in two curves from two sources. The reason of these congruent because the big number of coordinates but when zooming in this difference and finding the difference between the coordinates from the first and the second source and drawing this relationship between time and this difference it is clear that there is a marked difference between the coordinates values from two sources. For the entire study interval, the difference between the broadcast orbits and the precise orbits was determined at regular interval 15-minuts and initially calculated in Earth-Centered Earth-Fixed (ECEF) coordinates. Figures 5, 6 and 7 show the relationship between time horizontally in hours units and this difference as a y in meters for each of the three directions X, Y, Z in ECEF. To modelling these differences as well as predicting future coordinate values of satellite, a mathematical model (equation 23) was suggested to reach this task. From figures 5, 6, and 7 it can be seen that there is a strong periodicity which explains the cause of periodicity of the parameters to a certain extent. Since ephemeris error is a result of prediction of satellite positions, the size of this error will increase as one moves away from the reference epoch for ephemerides. Broadcast satellite position varied from precise positions and the deviation in x-coordinate is 31.21 to −31.05 meters over 60 hours. Similarly the error varied from 25.11 to -1.14 meters and 39.21 to -29.89 meters for y- and z-coordinates respectively. The next section presents in detail both the methodology and the step-by-step procedure used to apply Fourier series on historical time series to predict ECEF satellite coordinates. Next, the results of the validation of the proposed approach on the real data are presented.

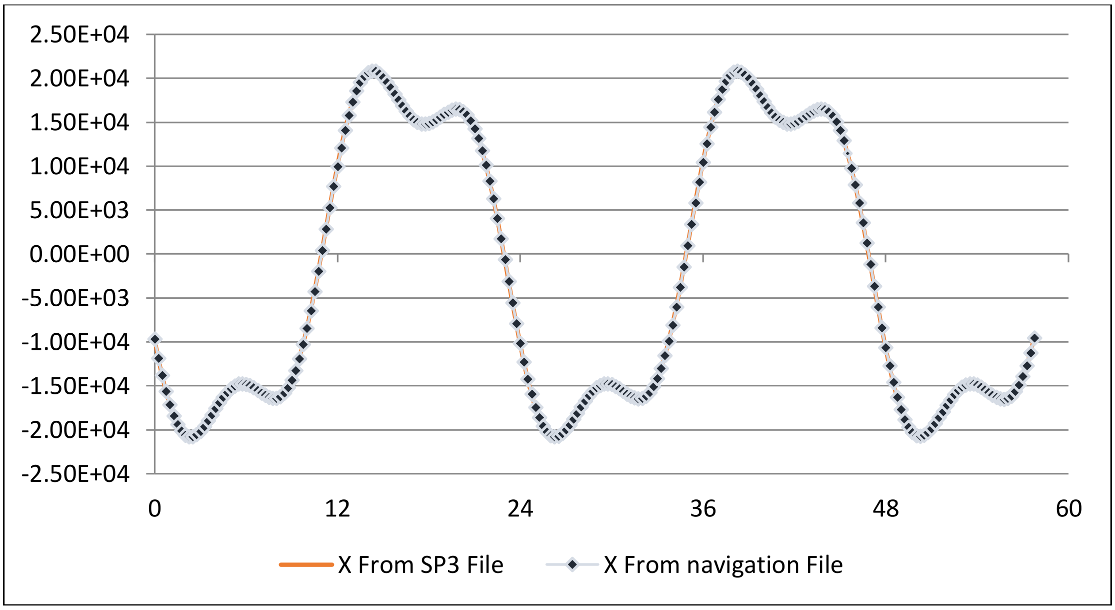

Figure 2. Relationship between time in hours as x axis and X_{ECEF} coordinates in km for satellite as y axis

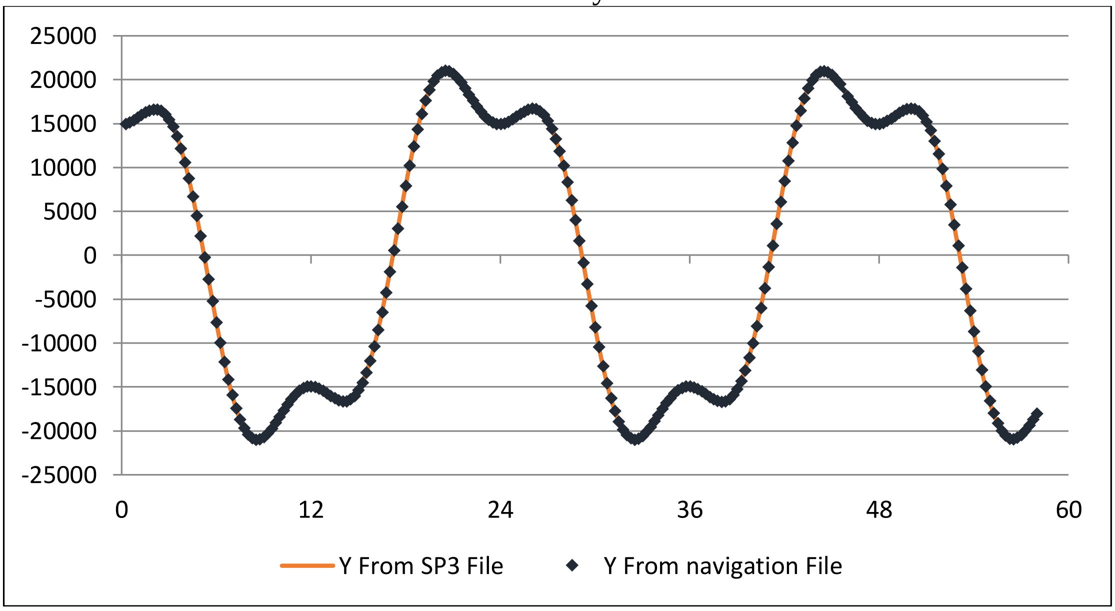

Figure 3. Relationship between time in hours as x axis and Y_{ECEF} coordinates in km for satellite as y axis

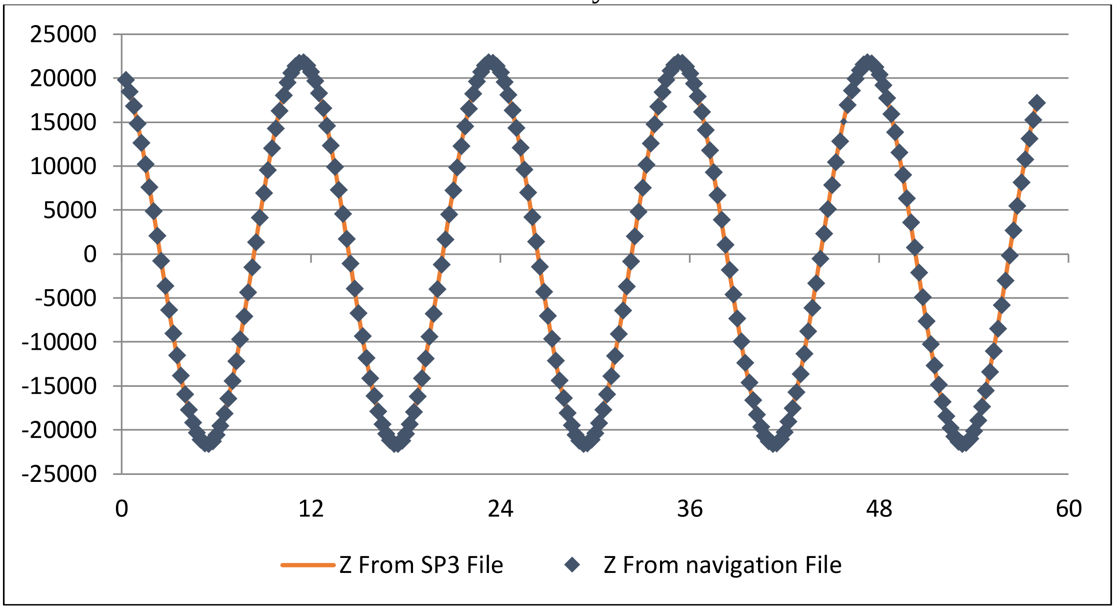

Figure 4. Relationship between time in hours as x axis and Z_{ECEF} coordinates in km for satellite as y axis

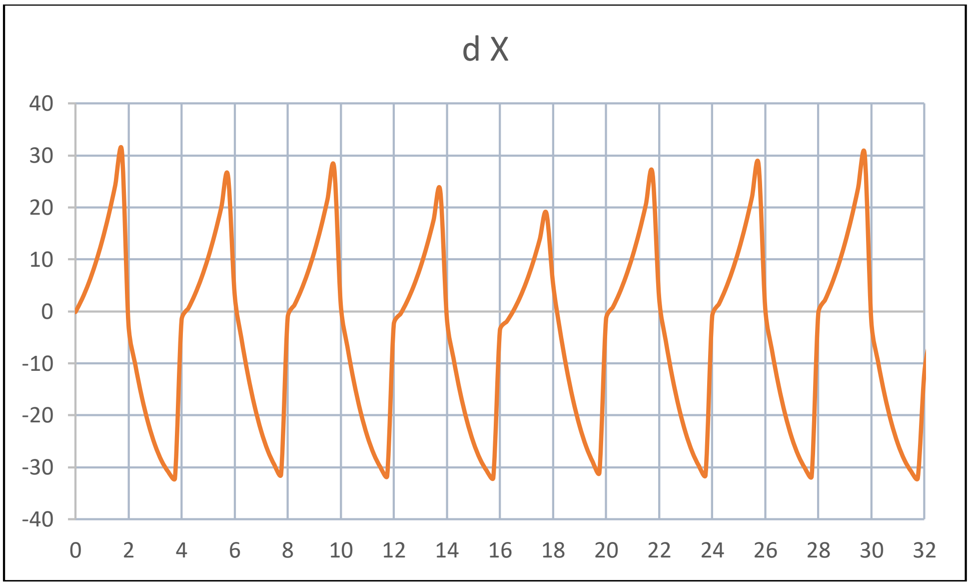

Figure 5. The difference between the precise and broadcast orbits in X_{ECEF} direction Horizontal axis is time in hours and vertical axis the difference in ms

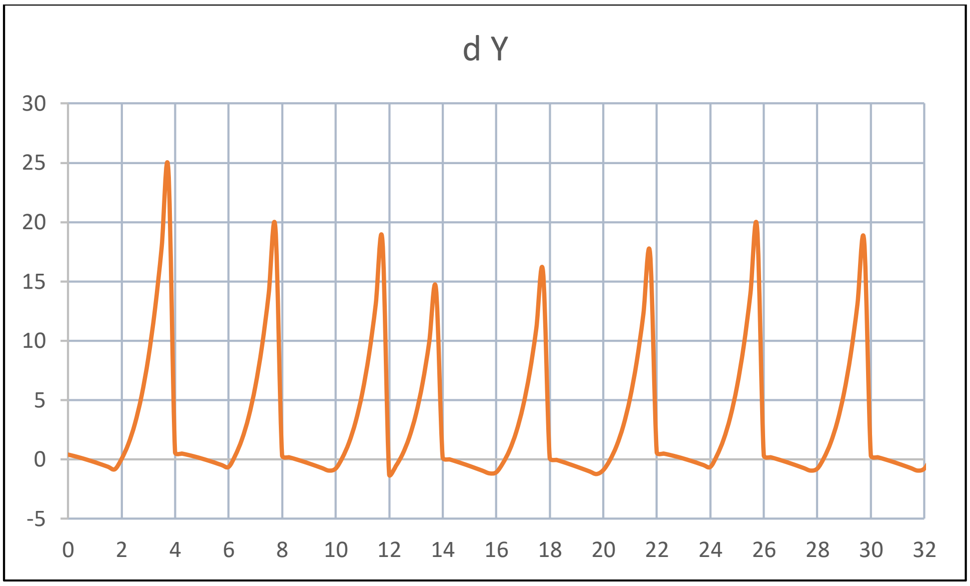

Figure 6. The difference between the precise and broadcast orbits in Y_{ECEF} direction
Horizontal axis is time in hours and vertical axis the difference in ms

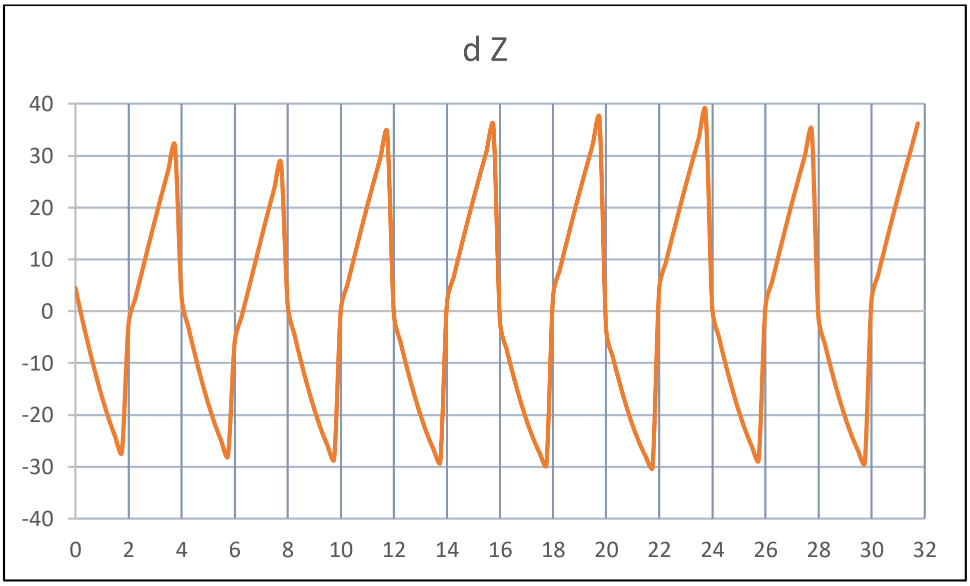

Figure 7. The difference between the precise and broadcast orbits in Z_{ECEF} direction
Horizontal axis is time in hours and vertical axis the difference in ms

3.1 The modelling and Prediction Algorithm

To characterize the cyclic signal, the Fourier series which is a set of sine and cosine functions can be used. The following expression expresses this function

$$F(t) = a_0 + \sum_{i=1}^{n} a_i \cos(i\,w\,t) + b_i \sin(i\,w\,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(22)}$$

Where a_0 represents a constant term in the model and is related with the $i = 0$, w is the fundamental frequency of the signal, n represents model terms number, and $1 \leq n \leq 8$. **[11]**

In this section of paper, many attempts were made to determine the best value of n. after these attempts, it is recommended that to take the value of n equal to 6 for X_{ECEF} coordinates and n equal to 6 for Y_{ECEF} and Z_{ECEF} coordinates and when n is set to 6 and rewritten equation 22 becomes as shown in equation 23.

$$F(t) = a_0 + a_1 \cos(t * w) + b_1 \sin(t * w) + a_2 \cos(2t * w) + b_2 \sin(2t * w) + a_3 \cos(3t * w) + b_3 \sin(3t * w) + a_4 \cos(4t * w) + b_4 \sin(4t * w) + a_5 \cos(5t * w) + b_5 \sin(5t * w) + a_6 \cos(6t * w) + b_6 \sin(6t * w) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(23)}$$

The best method to solve this model is the Gaussian method of the least squares. If there are n pairs of observations (t, F(t)), to determine the coefficient values it is required to minimize the sum of the squares of the deviations. Several tests were performed using the suggested Fourier algorithm with real broadcast and precise ephemerides. Based on these tests, the most suitable values of the unknown parameters are shown in table 7. All calculations and analyses were done with the data for satellite PRN#1.

Table 7: The values of predicted method coefficients

	Coeff. of X	Coeff. of Y	Coeff. of Z
a0	767.907758	5999.94243	15624.6829
a1	-1.503	-5055	-1.27E+04
b1	1.0038303	-9031	-2.34E+04
a2	-756.3	-2989	-8767
b2	-1176	5428.66411	1.33E+04
a3	0.69475988	2558.36268	6726.43044
b3	1.50164864	-85.23	547.151144
a4	-377.9	-406.5	-700.1
b4	446.451064	-531.4	-1604
a5	0.47066714	-13.17	-130.7
b5	-0.2559	83.914657	155.066285
a6	47.1819942		
b6	42.0343365		
w	0.13126849	0.26254657	0.26254409

GPS satellite coordinates were computed in the ECEF coordinate system using broadcast ephemerides data in order to modeling and prediction of GPS satellite orbits from navigation file. Due to the accuracy of the precise orbits are generally more than the broadcast orbits, the precise orbits were considered to be the truth, and any variation between the two is considered error in broadcast orbit. In order to verify the feasibility and accuracy of this method, three figures (figure 8, 9, and 10) were drawn between time in horizontal axis and predicted errors as y axis in meters for each of the three directions X, Y, Z in ECEF.

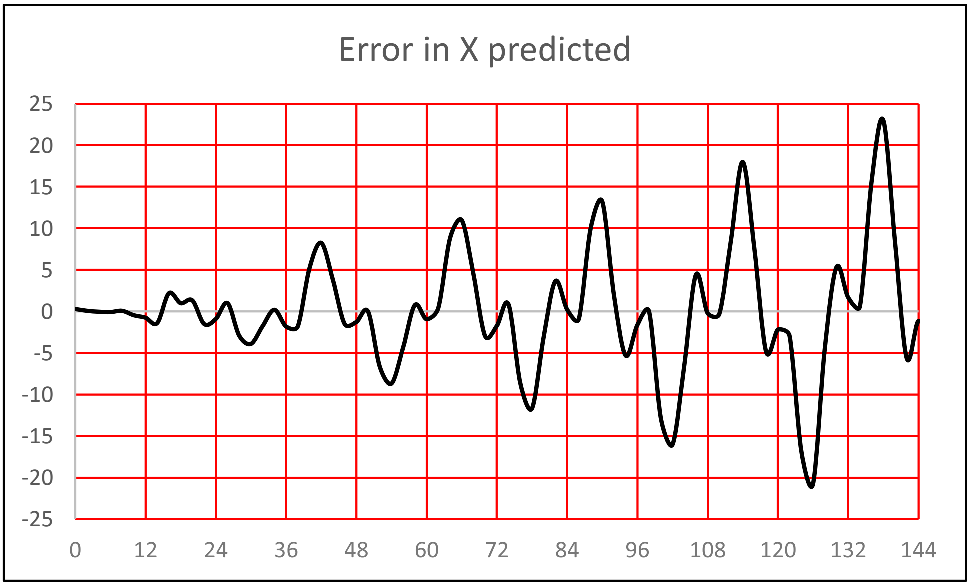

Figure 8. Errors in calculating orbit. Horizontal axis is time in hours and errors in predicted X_{ECEF} in ms is y axis

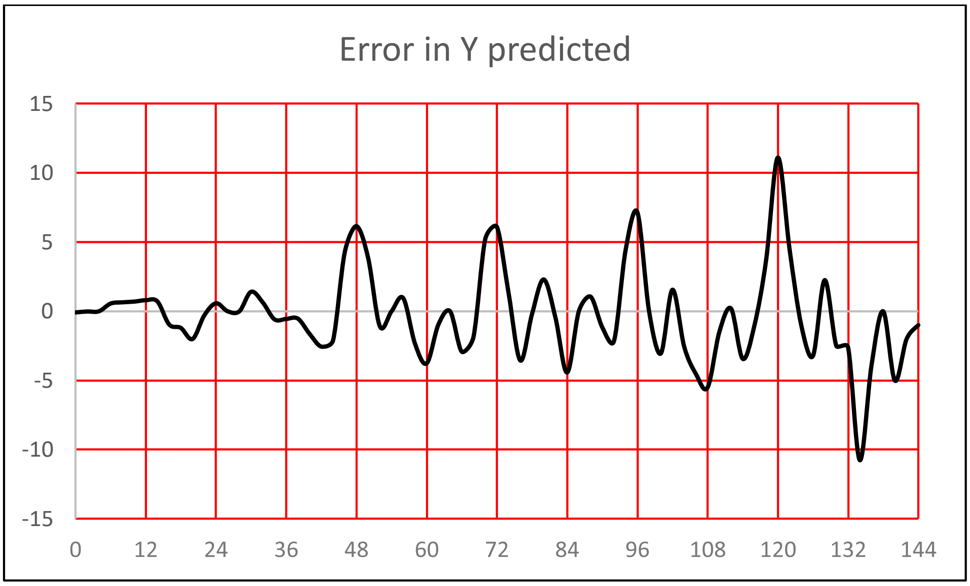

Figure 9. Errors in calculating orbit. Horizontal axis is time in hours and errors in predicted Y_{ECEF} in ms is y axis

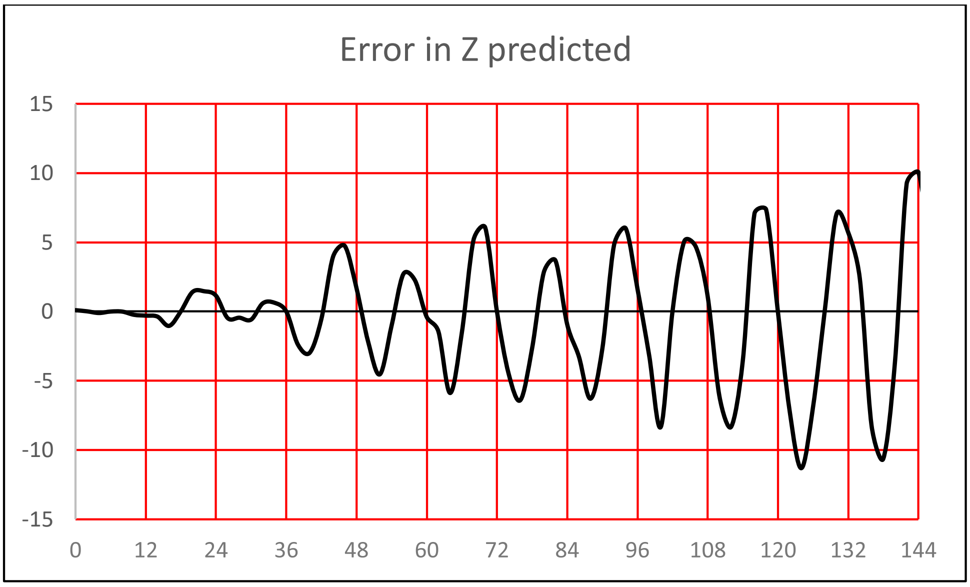

Figure 10. Errors in calculating orbit. Horizontal axis is time in hours and errors in predicted Z_{ECEF} in ms is y axis

From figure 8 to 10 it is clear that the errors in calculating the orbit are don't exceeds 35 cm for the first 12 hours and this errors in predicted orbit increased to sub-meter for the second 12 hours. This errors in predicted orbit are increased to be bigger than 5 m after 60 hours.

4. Conclusion

The parameters of GPS broadcast ephemeris broadcasted regularly once every two hours. The satellite orbit coordinate at whenever is calculated by reference to the reference time which is likewise called broadcast time and based on the analysis which is performed in this paper it very well may be presumed that:

- The calculated errors in ECEF coordinates from navigation file follow a certain trend.
- A new satellite orbit prediction method is proposed to predict satellite ECEF coordinates at some time in the future by using broadcast ephemeris history data. The accuracy and feasibility of the method are verified by abundant reliable data analyses.
- This paper presents an orbit prediction algorithm which can run on receivers whose network connections are nonexistent or in the absence of ground station support to enhance the accuracy especially in real-time calculation of GPS satellite orbit.
- The results of orbit prediction for GPS PRN#1 with the model described in this paper were show. From this results, it can see that the satellite’s position error limited to 35 cm for the first 12 hours and this errors in predicted orbit increased to sub-meter for the second 12 hours. This errors in predicted orbit are increased to be bigger than 5 m after 60 hours.
- With only analysis of the orbit parameters and avoiding the influence of other unknown factors, this method simply and effectively replaces the complex process of dynamic modeling which is common in autonomous orbit determination for the satellite orbit prediction.

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